Investor Sophistication and Capital Income Inequality ONLINE APPENDIX

Marcin Kacperczyk
Imperial College

Jaromir Nosal
Boston College

Luminita Stevens
FRB Minneapolis and University of Maryland

Abstract

This file contains supplementary material for the paper 'Investor Sophistication and Capital Income Inequality', by Kacperczyk, Nosal, and Stevens.

Contents

List of Figures

1 Data

Our model parametrization is based on the data from the Survey of Consumer Finances (SCF) from 1989 to 2013. For all computed statistics, we weigh all observations by the weights provided by the SCF (variable `42001`). Consistent with previous studies we drop farm owners.

1.1 Data Constructs

Participation Our measure of participation in financial markets includes individuals who satisfy at least one of the following criteria: (i) have a brokerage account (coded in variable `3923`), (ii) report a positive amount of stock holdings (variable `3915`), (iii) report holding non money market funds (coded as a positive balance in at least one of the variables: `3822`, `3824`, `3826`, `3828`, and `3830`; and also `7787` starting in survey year 2004), (iv) report positive holdings of bonds (coded as the sum of: `3906`, `3908`, `3910`, and additionally `7633`, `7634` starting in survey year 1992), (v) report dividends from their stock holdings (variable `5710`), (vi) report holding money funds (coded in variables: `3507`, `3511`, `3515`, `3519`, `3523`, `3527`).

As a robustness check, we also consider a measure of broad market participation that includes the above six, plus the condition that a household has equity in a retirement account. Specifically, we consider the criterion that (vii) a household reports that either the head or spouse or other family members have money in retirement accounts invested in equity. For survey years 1989 and 1992 it is coded in variable `3631` with values of 2 (stocks, mutual funds), 4 (combination of stocks, CDs and money market accounts, and bonds), 5 (combination of stocks and bonds), 6 (combination of CDs and money market accounts, and stocks). For survey years 1995, 1998, and 2001 it is coded in variable `3631` with values of 2, 4, 5, 6, or 16 (brokerage account/cash management account). For surveys starting in 2004, the coding shifts to variables `6555`, `6563`, or `6571` (head, spouse, other family members). For the 2004, 2007, and 2013 surveys, this means values 1 (all in stocks), 3 (split), or 5 (hedge fund) for at least one of the variables. For survey year 2010, this means answering 1, 3, 5, or 30 (mutual fund). Adding the category of -7 (other) to the above list does not change the results.

Capital Income To construct a measure of capital income, we sum up income from four sources: (i) dividend income (5710), (ii) income from non-taxable investments such as municipal bonds (5706), (iii) net gains or losses from mutual funds, sale of stocks, bonds, or real estate (5712), and (iv) other interest income (5708).

Wealth Measures Total wealth is a sum of financial and non-financial wealth as per Bucks, Kennickell, and Moore (2006). Financial wealth is a sum of: (1) holdings in non money funds (sum of balance in variables: 3822, 3824, 3826, 3828, 3830, and also 7787 starting in survey year 2004), (2) bond holdings balance (the sum of: 3906, 3908, 3910, and also 7633, 7634 starting in survey year 1992), (3) balance of directly held stocks (variable 3915), (4) cash value of life insurance (4006), (5) other financial assets (future royalties, money owed to households, etc. in variable 4018), (6) balances in individual retirement accounts of all family members (variables 6551-6554, 6559-6562, 6567-6570, 6756, 6757, 6758), (7) value of certificates of deposit (3721), (8) cash value of annuities, trusts, or managed accounts (6577, 6587), (9) value of savings bonds (3902), (10) value of liquid assets (checking accounts 3506, 3510, 3514, 3518, 3522, 3526, 3529, cash or call money accounts 3930, savings and money market accounts 3730, 3736, 3742, 3748, 3754, 3760).

Non-financial wealth is a sum of: (1) value of vehicles, including motor homes, RVs, motorcycles, boats, and airplanes less the amount still owed on the financing loans for these vehicles (8166+8167+8168+8188-2218-2318-2418-7169+2506+2606-2519-2619+2623-2625), (2) value of business in which a household has either active or nonactive interest (value of active business is calculated as net equity if business was sold today plus loans from the household to the business minus loans from the business to the household, plus value of personal assets used as collateral for business loans; value of non-active business is the market value; the formula used (for the 2004 SCF) is 3129+3229+3329+3335+8452+8453+3408+3412+3416+3420 +3424+3428+3124+3224+3324-(3126+3226+3326) plus 3121+3221+3321 (variables have different numbers pre-1995; some variables are not reported in 2010 and 2013 anymore), (3) value of houses and mobile homes/sites owned (604+614+623+716), (4) value of other real estate owned: vacation homes (2002) and owned share of other property (1706*1705+1806*1805+1906*1905 divided by 10000), (5) the value of other non-residential real estate net of mortgages and other loans taken out for investment in real estate (2012-2016), (6) other non-financial assets, such as artwork, precious metals, antiques, oil and gas leases, futures contracts, future proceeds from a lawsuit or estate that is being settled, royalties, or something else (4022+4026+4030).

Wage Income and Total Income For labor income and total income, we use the SCF responses to questions 5702 (income from wages and salaries) and 5729 (income from all sources). The difference between the two, apart from capital income, consists of social security and other pension income, income from professional practice, business or limited partnerships, income from net rent, royalties, trusts and investment in business, unemployment benefits, child support, alimony and income from welfare assistance programs.

1.2 Participation

In Figure 1, we present the time series of our two measures of participation. The series *Participation* follows our benchmark definition above, while *Participation + Retirement* is a broader measure that also includes individuals who participate in equity through retirement accounts.

Our participation measure changes from 32% in 1989 to a high of 40% in 2001, and down to 28% in 2013. When we additionally include participation through retirement accounts, the dynamics are very similar, except that the levels get shifted upwards. The participation level is around 35% in 1989, peaks at 44% in 2001, and goes down to 37% in 2013.

Even though both measures of participation exhibit considerable variation over time (although without any particular trend), as we point out in the paper, financial wealth inequality in the SCF data set is entirely concentrated within our participating group. Figure 5 in the paper, reproduced in Figure 2 below, presents financial wealth inequality between

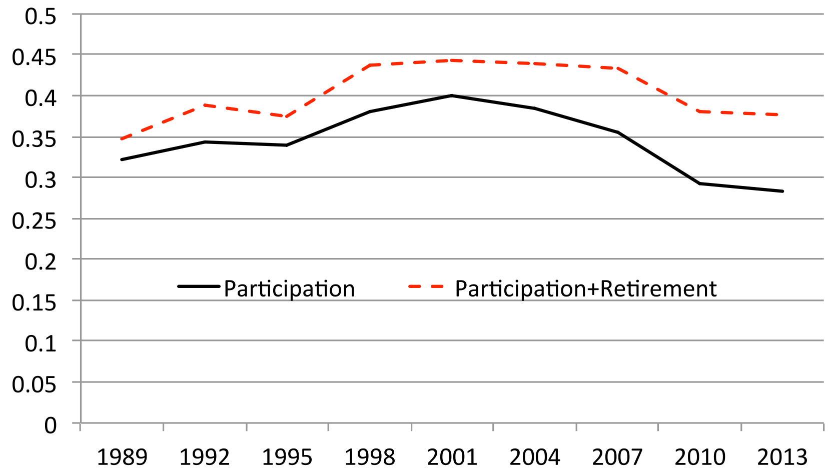

Figure 1: Financial markets participation in the SCF.

(i) top decile and bottom half of our participating group ('Sophisticated/Unsophisticated'), (ii) bottom half of participants and non-participants ('Unsophisticated/Non-participants'), and (iii) bottom decile of participants and non-participants. Financial wealth inequality between the unsophisticated and non-participants, as well as bottom participants and non-participants exhibits no trend and the ratios are stable around 1. Additionally, in Figure 8 in the paper, we show that all of the growth in financial wealth inequality in our participating group can be accounted for by retained capital income. These two points suggest that the participating group is the relevant subsample to study capital income inequality.

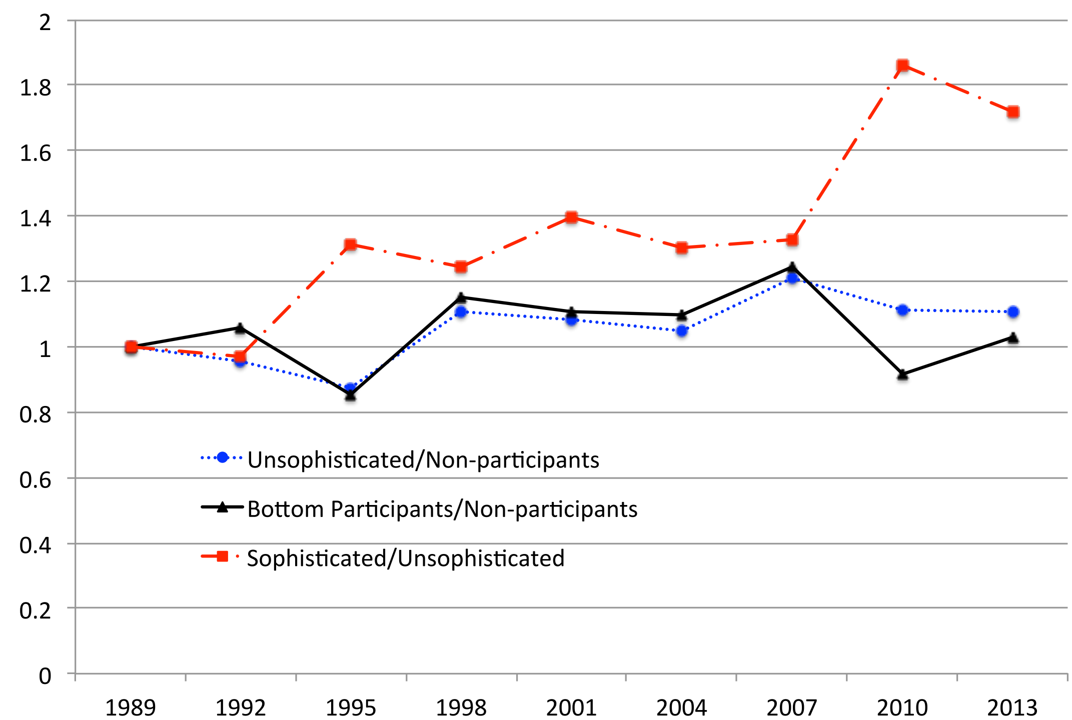

Figure 2: Extensive and intensive margins in capital income inequality.

1.3 Capital Income

Inequality Our measure of inequality is the mean income in the top decile of the wealth distribution relative to the mean income in the bottom half (*of participants*). Figure 3 presents the evolution of capital income inequality in the SCF. Figure 4 presents the normalized evolution of capital income inequality using the benchmark definition of participation as well as Participation+Retirement.

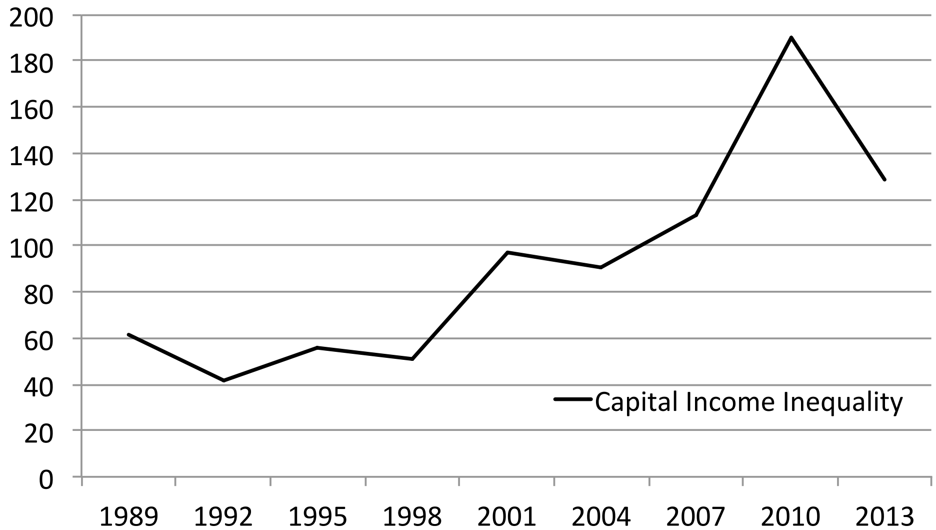

Figure 3: Capital income inequality.

Passive Investment Policies We also study whether capital income differences are an outcome of time-varying *market returns* combined with *passive* buy-and-hold household strategies. It is possible that some households (the wealthy) hold a larger share of their wealth in stocks, which gives them higher returns by the mere fact that stocks outperform bonds. In Figure 5 we plot, for each year, the past 15-year cumulative return on holding the aggregate index of the U.S. stock market.[1] We contrast this return with that of a household exclusively holding bonds (with a gross return of 1).

The cumulative return on the passive strategy exhibits a declining trend, which implies that if investors used the passive strategy and the only difference was how much money they

[1]The patterns we document are essentially the same for other choices of the horizon: 5, 10, or 20 years.

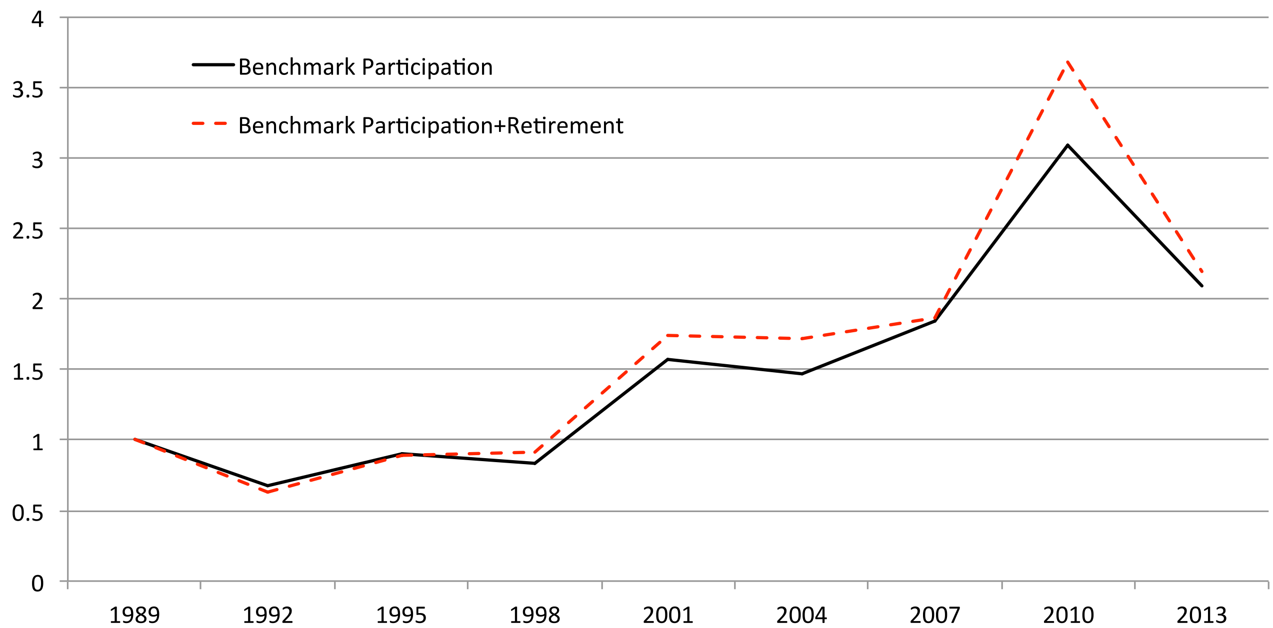

Figure 4: Capital income inequality for different measures of participation.

hold in the stock market versus bonds, then we should observe a declining trend in capital income inequality, as the gross return on the market converges to the gross return on bonds. This exercise highlights the importance of active decisions of when to enter and exit the stock market.

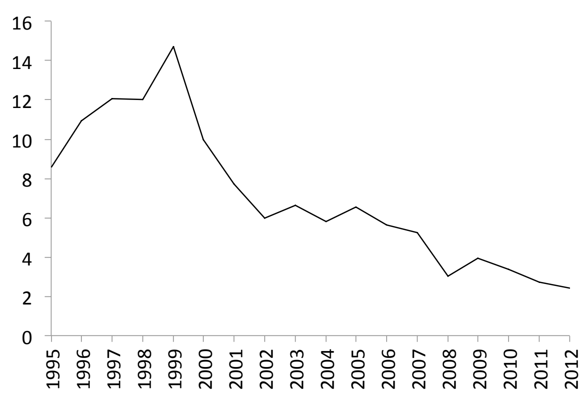

Figure 5: Cumulative market return on a 15-year passive investment in the U.S. stock market.

1.4 Survey of Consumer Finances: Descriptive Statistics

To complete the characterization of the participating and non-participating groups in the SCF, Table 1 presents summary statistics for the 1989 and 2013 surveys. As expected, participants in financial markets tend to be wealthier, older and more educated. Within the participating group, the top 10% of participants also have higher financial wealth, are older, and more educated. However, Panel I of the table shows that the growth in financial wealth inequality is concentrated almost exclusively within the participating group, consistent with the trends in Figure 4. First, in the cross-section, the financial wealth of the bottom 50% of participants is only twice that of the non-participants; conversely, in 1989 the top 10% of participants has financial wealth that is 38 times larger than that of the bottom 50%. Second, between 1989 and 2013, financial wealth inequality within the participants group grew by 67%, while inequality across groups grew by mere 12%. Panels II through IV of Table 1 summarize the inequality in capital, labor, and total income for participants and non-participants. The same pattern that emerged with respect to financial wealth inequality also applies to labor and total income inequality: both the level and the growth of inequality have been concentrated within the group of participants.

Panels V through IX of the table explore potential drivers of the growth in inequality between the top 10% and the bottom 50% of participants. First, top participants earn more capital income per dollar of financial wealth, a crude measure of their rates of return (Panel V). Second, top participants hold a much smaller fraction of their financial wealth in liquid assets (Panel VI). In turn, bottom participants start out with a higher share (33% versus 21%) and also grow the fraction of financial wealth held in liquid assets significantly (from 33% in 1989 to 46% in 2013). This type of portfolio composition shift towards lower risk liquid assets for the bottom participants is consistent with our information-based mechanism. Third, top participants also have higher educational attainment and are much more likely to have brokerage accounts (Panels VII and VIII), consistent with their having a higher degree of financial sophistication. The data, however, also show a significant increase in access to brokerage accounts for the bottom participants (from 16% in 1989 to 35% in 2013). This fact,

Table 1: Investor Characteristics in the SCF

	1989	2013
I. Financial Wealth		
Top 10%/Bottom 50% of Participants	38	64
Bottom 50%/Non-participants	2.0	2.2
II. Capital Income		
Top 10%/Bottom 50% of Participants	61	129
Bottom 50%/Non-participants	-	-
III. Wages and Salaries Income		
Top 10%/Bottom 50% of Participants	3.3	5.6
Bottom 50%/Non-participants	1.3	1.4
IV. Total Income		
Top 10%/Bottom 50% of Participants	8.3	11.4
Bottom 50%/Non-participants	1.3	1.3
V. Capital Income/Financial Wealth		
Top 10% of Participants	10.7%	4.6%
Bottom 50% of Participants	6.7%	2.3%
VI. Liquid Assets/Financial Wealth		
Top 10% of Participants	21%	19%
Bottom 50% of Participants	33%	46%
Non-participants	49%	74%
VII. Has brokerage account		
Top 10% of Participants	64%	83%
Bottom 50% of Participants	16%	36%
VIII. % with college		
Top 10% of Participants	57%	60%
Bottom 50% of Participants	49%	51%
Non-participants	46%	50%
IX. Age (years)		
Top 10% of Participants	57	60
Bottom 50% of Participants	49	51
Non-participants	46	50

Source: SCF. Capital income/Financial wealth is the ratio of average capital income to the average financial wealth in each group. Percent with college is the fraction of individuals with 16 or more years of schooling. See the Online Appendix for complete definitions.

along with evidence that transaction costs on brokerage accounts have been trending down (French (2008)), suggests that the costs of accessing and transacting in financial markets are an unlikely explanation for the observed rise in capital income inequality. If anything, the improved access to financial markets should generate lower inequality, in the absence of informational heterogeneity. Finally, while top participants are on average older, there are no time-series dynamics to the age difference that could explain the observed capital income dynamics (Panel IX).

2 Theory: Detailed Proofs and Additional Results

2.1 The Value of Prices

In our analysis in the paper, we have presented the information acquisition problem in terms of a constraint on information obtained through private signals alone, excluding the information contained in prices. When some investors acquire information through private signals, prices become informative about assets payoffs, because they reflect the demand of these privately informed investors. In the literature on portfolio choice with exogenous signals, investors are often assumed to learn about payoffs not only from their private signals, but also from equilibrium prices, which aggregate the information of all investors in the market (e.g., Admati (1985)). Would investors with an endogenous signal choice have an incentive to allocate any capacity to learning from prices? We show that if the information contained in prices is costly to process, then prices are an inferior source of information compared with private signals.

We consider the signal choice of an individual investor, taking the choices of all other investors as given by the equilibrium obtained in Section 2.2. Processing information through either prices or private signals consumes the investor's capacity. Hence, whatever the source of information, the investor cannot acquire a total quantity beyond her capacity K_j.

Proposition 1 (**Prices**). *If learning about prices consumes capacity, then the capacity-constrained investor chooses to devote all her capacity to learning about payoffs through* *private signals on asset payoffs, rather than devoting any capacity to learning from prices.*

Intuitively, prices represent an indirect way of learning about asset payoffs, which are ultimately what investors seek to learn. Our proof follows the logic of Kacperczyk et al. (2015) although it is derived for a different information structure and extended to include the case in which the information content of prices is not processed perfectly.

If processing the noise trader shock also consumes capacity then Proposition 1 implies that investors will not allocate any capacity to learning about the supply shock, ν_i. Learning about the activity of noise traders is not useful *unless* that information is combined with information processed from prices. It is only the joint information on both variables that informs investors about payoffs.

Proof of Proposition 1. We consider the choice of an individual investor, taking the choices of all other investors as given, characterized by the solution in the main text.

Case A. First, we consider the case in which the investor treats the price as any other random variable that cannot be processed perfectly for free. Suppose that the investor allocates capacity to learning the price of asset i. This investor will observe a compressed representation of the price, s_{ji}^p, that is the result of the decomposition $p_i = s_{ji}^p + \varepsilon_{ji}$,with $s_{ji}^p \sim \mathcal{N}\left(\overline{p}_i, \sigma_{spji}^2\right), \varepsilon_{ji} \sim \mathcal{N}\left(0, \sigma_{\varepsilon ji}^2\right)$, and $\sigma_{pi}^2 = \sigma_{spji}^2 + \sigma_{\varepsilon ji}^2$. The amount of capacity consumed by the price signal is

$$I\left(p_i; s_{ji}^p\right) = \frac{1}{2}\log\left(\frac{\sigma_{pi}^2}{\sigma_{\varepsilon ji}^2}\right).$$

The quantity of information about *payoffs* that is conveyed by the price signal is

$$I\left(z_i; s_{ji}^p\right) = H\left(z_i\right) + H\left(s_{ji}^p\right) - H\left(z_i, s_{ji}^p\right) = \frac{1}{2}\log\left(\frac{\sigma_i^2\sigma_{spji}^2}{\left|\Sigma_{z_i sp_{ji}}\right|}\right),$$

where $\left|\Sigma_{z_i sp_{ji}}\right|$ is the determinant of the variance-covariance matrix of z_i and s_{ji}^p. Using the fact that z_i and s_{ji}^p are conditionally independent given prices, $Cov\left(z_i, s_{ji}^p\right) = Cov\left(z_i, p_i\right) Cov\left(p_i, s_{ji}^p\right)/\sigma_{pi}^2$. Using the solution for equilibrium prices, $Cov\left(z_i, p_i\right) = b_i\sigma_i^2$. Using the signal structure, $Cov\left(p_i, s_{ji}^p\right) = \sigma_{spji}^2$. Hence $Cov\left(z_i, s_{ji}^p\right) = b_i\sigma_i^2\sigma_{spji}^2/\sigma_{pi}^2$. The determinant becomes

$\left|\Sigma_{z_i s p_{ji}}\right| = \sigma_i^2 \sigma_{spji}^2 \left(\frac{\sigma_{pi}^2 \sigma_{pi}^2 - b_i^2 \sigma_i^2 \sigma_{spji}^2}{\sigma_{pi}^2 \sigma_{pi}^2}\right)$, so that

$$I\left(z_i; s_{ji}^p\right) = \frac{1}{2}\log\left(\frac{\sigma_{pi}^2}{c_i^2 \sigma_{xi}^2 + \frac{b_i^2 \sigma_i^2}{\sigma_{pi}^2}\sigma_{\varepsilon ji}^2}\right).$$

Next, we show that $I\left(z_i; s_{ji}^p\right) \leq I\left(p_i; s_{ji}^p\right)$. Suppose not. Then, in order for the reverse inequality to hold, it must be the case that

$$c_i^2 \sigma_{xi}^2 < \left(1 - \frac{b_i^2 \sigma_i^2}{\sigma_{pi}^2}\right)\sigma_{\varepsilon ji}^2 \quad \Leftrightarrow \quad \sigma_{pi}^2 < \sigma_{\varepsilon ji}^2,$$

which is a contradiction. Hence, $I\left(z_i; s_{ji}^p\right) \leq I\left(p_i; s_{ji}^p\right)$, with equality if and only if $\sigma_{pi}^2 = \sigma_{\varepsilon ji}^2$, which occurs only if $I\left(p_i; s_{ji}^p\right) = 0$. Hence for any positive capacity dedicated to the price signal, the effective amount of information about the payoff is less than the capacity consumed in order to receive the signal.

Case B. Next, we consider the case in which the price itself is a perfectly observed signal that nonetheless consumes capacity. Suppose that the investor uses capacity to learn from p_i, and let posterior beliefs about z_i conditional on p_i be denoted by y_i. Then $y_i \sim \mathcal{N}\left(\overline{y}_i, \sigma_{yi}^2\right)$, with

$$\overline{y}_i = \sigma_{yi}^2 \left[\frac{1}{\sigma_i^2}\overline{z}_i + \frac{b_i^2}{c_i^2 \sigma_{xi}^2} z_i - \frac{b_i}{c_i \sigma_{xi}^2}\left(x_i - \overline{x}_i\right)\right]$$

$$\frac{1}{\sigma_{yi}^2} = \frac{1}{\frac{1}{\sigma_i^2} + \frac{b_i^2}{c_i^2 \sigma_{xi}^2}}.$$

The information contained in the price of asset i is $I\left(z_i; p_i\right) = \frac{1}{2}\log\left(\frac{\sigma_i^2}{\sigma_{yi}^2}\right)$. Using the solution for equilibrium prices, this variance is given by

$$\sigma_{yi}^2 = \frac{\sigma_i^2}{1 + \left(\frac{\phi m_i}{\rho \sigma_i \sigma_{xi}}\right)^2}.$$

We next demonstrate that the investor's ex-ante expected utility is higher when allocating all her capacity to learning from private signals than when allocating at least a portion of her capacity to learning from prices, owing to strategic substitutability. The investor's objective is to maximize

$$\widetilde{E}_{1j}\left[U_{2j}\right]=\frac{1}{2\rho}\sum_{i=1}^{n}\left(\frac{\widetilde{V}_{ji}+\widetilde{R}_{ji}^{2}}{\widetilde{\sigma}_{ji}^{2}}\right)\ s.t.\ \prod_{i=1}^{n}\left(\frac{\sigma_{i}^{2}}{\widetilde{\sigma}_{ji}^{2}}\right)\leq e^{2K_{j}},$$

where $\widetilde{R}_{ji}$ and $\widetilde{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, $(\widetilde{\mu}_{ji}-rp_i)$, $\widetilde{\mu}_{ji}$ and $\widetilde{\sigma}_{ji}^2$ denote the mean and variance of the investor's posterior beliefs about the payoff z_i, and the tilde indicates that these variables are computed under a signalling mechanism that allows for learning from prices.

Suppose that the investor uses capacity to learn from p_i, and let posterior beliefs about z_i conditional on p_i be denoted by y_i. Then, the investor designs a signal conditional on the information obtained from the price, $y_i=\widetilde{s}_{ji}+\widetilde{\delta}_{ji}$, where we maintain the same two independence assumptions that were used in setting up the private signal in the absence of learning from the price. Under this signal structure, the ex-ante mean is the same, regardless of whether the investor learns from p_i or not: $\widetilde{R}_{ji}=\overline{z}_i-r\overline{p}_i$. The ex-ante variance of expected excess returns is given by $\widetilde{V}_{ji}=Var_{1j}\left(\widetilde{\mu}_{ji}\right)+r^2\sigma_{pi}^2-2rCov_{1j}\left(\widetilde{\mu}_{ji},p_i\right)$. Using the formula for partial correlation and exploiting the fact that signals and prices are conditionally independent given beliefs, $Cov_{1j}\left(\widetilde{\mu}_{ji},p_i\right)=Cov_{1j}\left(\widetilde{\mu}_{ji},y_i\right)Cov_{1j}\left(y_i,p_i\right)/\sigma_{yi}^2$. Using the signal structure, $Cov_{1j}\left(\widetilde{\mu}_{ji},y_i\right)=Var\left(\widetilde{s}_{ji}\right)$, $Var\left(\widetilde{s}_{ji}\right)=\sigma_{yi}^2-\widetilde{\sigma}_{ji}^2$, and using equilibrium prices, $Cov_{1j}\left(y_i,p_i\right)=b_i\sigma_i^2$ and $Cov_{1j}\left(\widetilde{\mu}_{ji},p_i\right)=b_i\sigma_i^2-b_i\sigma_i^2\widetilde{\sigma}_{ji}^2/\sigma_{yi}^2$. Hence, $\widetilde{V}_{ji}=\left(1-2rb_i\right)\sigma_i^2+r^2\sigma_{pi}^2-\left(\sigma_i^2-\sigma_{yi}^2\right)-\left[1-2rb_i\left(\frac{\sigma_i^2}{\sigma_{yi}^2}\right)\right]\widetilde{\sigma}_{ji}^2$, if the investor learns from p_i.

Conversely, if the investor does not allocate any capacity to learning from prices, $V_{ji}=\left(1-2rb_i\right)\sigma_i^2+r^2\sigma_{pi}^2-\left(1-2rb_i\right)\widetilde{\sigma}_{ji}^2$, where we have used the fact that the information constraint implies that the investor's posterior variance, here denoted by $\widetilde{\sigma}_{ji}^2$, is the same in both cases. Both cases imply a corner solution, with the investor allocating all capacity to learning about a single asset. The remaining question is: will the investor allocate any capacity to learning from the price, or will she use all capacity on the private signal? It can be easily seen that for any positive level of capacity allocated to the price signal, $V_{ji}>\widetilde{V}_{ji}$. Hence, the investor's ex-ante utility is lower when she devotes any positive amount of capacity to learning from prices. Learning from prices increases the covariance between the investor's posterior beliefs and equilibrium prices, thereby reducing the investor's excess returns. This case is similar to that of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2015), who show that prices are an inferior source of information in a portfolio choice model with an additive constraint on the sum of signal precisions.

Hence, regardless of the informativeness of prices relative to the investor's capacity, the investor is always better off learning through signals that provide information directly on the payoffs. In our framework, prices lose their special role as publicly available signals. $\square$

2.2 Endogenous Capacity Choice

Below, we provide a numerical example of an endogenous capacity choice outcome in a model in which wealth heterogeneity matters for endogenous capacity choice. In particular, we assume that investors have identical CRRA preferences with IES coefficient γ, and differ in terms of their beginning of period wealth. Then, for each investor j, the absolute risk aversion coefficient is a function of wealth W_j, given by

$$A(W_j) = \gamma / W_j.$$

Locally, we map this into absolute risk aversion differences in a mean-variance optimization model by setting the coefficient ρ_j for investor j equal to $A(W_j)$. These differences in absolute risk aversion in the model imply differences in the size of the risky portfolio, and hence different gains from investing wealth in purchases of information capacity.

In particular, for a given cost of capacity given by the function $f(K)$, each investor type is going to choose the amount of capacity to maximize the ex-ante expectation of utility:

$$\frac{1}{2\rho_j} \sum_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{ij}^2} G_i - f(K_j),$$

where, in equilibrium, G_i is a function of the distribution of individual capacity choices of investors, but not of individual capacity choices, and $\widehat{\sigma}_{ij}^2 = \sigma_i^2 e^{-2K_j}$ if the investor learns about asset i.

The gain from increasing wealth is given by the benefit of increasing the precision of information for the asset that the investor is learning about. Since all actively traded assets have the same gain in equilibrium, we can express the marginal benefit of increasing capacity in terms of the gain of the highest volatility asset (asset 1), $\frac{1}{2\rho_j} e^{2K_j} G_1$, and then the optimization problem for capacity choice can be expressed as

$$\max_{K} \left\{ \frac{1}{2\rho_j} e^{2K} G_1 - f(K) \right\}. \tag{1}$$

Assumption 1 below ensures an interior solution to (1) exists.

Assumption 1. The following statements hold:
(i) For all j, $\frac{G_1}{\rho_j} - f'(0) > 0$, where G_1 is evaluated at $K_j = 0$ for all j,
(ii) There exists $\underline{K} > 0$, such that for all j and for all $K > \underline{K}$, $2\frac{G_1}{\rho_j} e^{2K} - f''(K) < 0$,
(iii) There exists $\bar{K} > 0$ such that for all j and for all $K > \bar{K}$, $\frac{G_1}{\rho_j} e^{2K} - f'(K) < 0$.

Numerical example Assume that the cost function is of the form: $f(K) = e^{aK}$. Under Assumption 1, the optimal choice of K for agent j is implicitly defined by:

$$\frac{G_1(\{\bar{K}_j\})}{\rho_j} = a e^{(a-2)K},$$

where we make the dependence of G_1 on the distribution of capacities explicit. Clearly, for any $a > 2$, the higher wealth investors (implying lower ρ_j) will choose higher capacity levels. However, because of the dependence of G on equilibrium capacity choices, to quantify the differences we need to solve the equilibrium fixed point of the model.

Figure 6 presents the ratio of capacities as a function of the cost parameter of capacity, a, for different values of the absolute risk aversion coefficient of the wealthy ρ_1 (which maps into different common relative risk aversion coefficients γ). The inequality in capacity exhibits a U-shape. First, if the cost of capacity is small, then the equilibrium inequality in capacity grows without bound, as the wealthier accumulate infinite capacity (faster than the less wealthy). For higher values of the cost of capacity, inequality exhibits a growing trend as the cost increases, very quickly approaching values in excess of 38, our benchmark value. It should be noted that even for the high values of the cost parameter, the overall cost relative to gain, $f(K_j)/\frac{1}{2\rho_j}e^{2K_j}G_1$, is relatively small, less than 1% for the wealthy and less than 6% for the less wealthy.

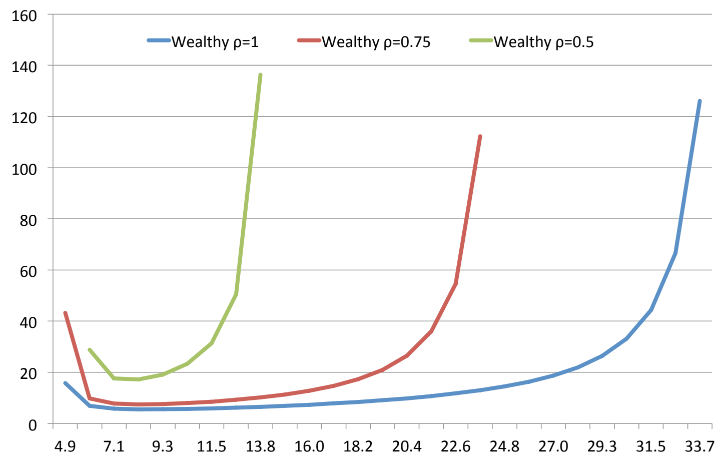

Figure 6: Inequality in information capacity (K_1/K_2) as a function of a and absolute risk aversion coefficient of the wealthy.

2.3 CRRA Utility Specification

Here, we solve the main investment problem of maximizing the expected utility of wealth, where the utility function is CRRA with respect to end of period wealth:

$$\max E\frac{W^{1-\rho}}{1-\rho} \tag{2}$$

where $\rho \neq 1$. Generally, for our specification of the payoff process, i.e. $z \sim \mathcal{N}(\bar{z}, \sigma_i^2)$, wealth next period is

$$W_{t+1} = r(W_t - \sum_i p_i q_i) + \sum_i q_i z_i$$

which has a Normal distribution if z_i's are Normal. In order to analytically express the expectation in (2), we start by expressing wealth as $W' = We^{\log\{[r(1-\sum p\frac{q}{W})+\sum \frac{q}{W}z]\}}$, and then use an approximation of the log return.

Approximation To approximate $\log\{[r(1-\sum p\frac{q}{W})+\sum\frac{q}{W}z]\}$, define

$$f(z-rp)\equiv\log[r+\frac{1}{W}\sum pq\frac{z-rp}{p}].$$

In the above equation, the term z is the only unknown stochastic term. Its Taylor approximation is

$$f(z-rp)=f(\bar{z}-rp)+f'(\bar{z}-rp)(z-\bar{z})+\frac{1}{2}f''(\bar{z}-rp)(z-\bar{z})^2+o(z-rp)$$

where in the above,

$$f'=\frac{1}{r+\frac{1}{W}\sum q(\bar{z}-rp)}\frac{q}{W},$$

$$f''=-\frac{1}{(r+\frac{1}{W}\sum q(\bar{z}-rp))^2}\frac{q^2}{W^2},$$

$$f'''=2\frac{1}{(r+\frac{1}{W}\sum q(\bar{z}-rp))^3}\frac{q^3}{W^3}.$$

With these formulas in hand, the approximation is

$$f(z-rp)=\log[r+\frac{1}{W}\sum q(\bar{z}-rp)]+\frac{1}{r+\frac{1}{W}\sum q(\bar{z}-rp)}\frac{q}{W}(z-\bar{z})$$

$$-\frac{1}{2}\frac{1}{(r+\frac{1}{W}\sum q(\bar{z}-rp))^2}\frac{q^2}{W^2}(z-\bar{z})^2$$

Denote

$$r+\frac{1}{W}\sum q(\bar{z}-rp)\equiv R(q)$$

Then we can write

$$f(z-rp)=\log[R(q)]+\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}\frac{1}{R(q)^2}\frac{q^2}{W^2}(z-\bar{z})^2,$$

and

$$(e^{\log(f(z-rp))})^{1-\rho}=e^{(1-\rho)(\log[R(q)]+\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}\frac{1}{(R(q))^2}\frac{q^2}{W^2}(z-\bar{z})^2)}$$

$$= (R(q))^{1-\rho} e^{(1-\rho)\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}(1-\rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}(z-\bar{z})^2}$$

We are interested in the object $e^{(1-\rho)\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}(1-\rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}(z-\bar{z})^2}$ from the above expression. First, we approximate the term $(z-\bar{z})^2$ by its expected volatility, $\sigma_{\delta i}^2$, to get

$$e^{(1-\rho)\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}(1-\rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}\sigma_{\delta i}^2}$$

As an approximation point, we pick $\bar{z}$, which gives a constant $R(q)$, and then

$$\log EW^{1-\rho} = const. \times \log E e^{(1-\rho)\frac{1}{R(q)}\frac{q}{W}(z-\bar{z})-\frac{1}{2}(1-\rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}\sigma_{\delta i}^2} \tag{3}$$

where the variable in the exponent is Normal, with mean (ignoring constants) $\sum q_i(\hat{\mu}_i - \bar{z}_i)$ and variance equal to $\sum q_i^2 \sigma_{\delta i}^2$. Then,

$$\log EW^{1-\rho} = const. \times (1-\rho)\left\{\frac{1}{R}\sum \frac{q}{W}(\hat{\mu}_i - \bar{z}_i) + (1-\rho)\frac{1}{W^2R^2}\frac{1}{2}\sum q_i^2\sigma_{\delta i}^2 \right.$$

$$\left. -\frac{1}{2}\frac{1}{W^2R^2}\sum q_i^2\sigma_{\delta i}^2\right\}$$

which gives

$$\log EW^{1-\rho} = const. \times (1-\rho)\left\{\frac{1}{R}\sum \frac{q}{W}(\hat{\mu}_i - \bar{z}_i) - \rho\frac{1}{W^2R^2}\frac{1}{2}\sum q_i^2\sigma_{\delta i}^2\right\}$$

Interior minimum (which maximizes $EW^{1-\rho}/(1-\rho)$) is

$$q_i = \frac{1}{\rho}\frac{\hat{\mu}_i - rp}{\sigma_{\delta i}^2}(Wr).$$

Plugging in gives:

$$U = \frac{1}{1-\rho}W^{1-\rho}r^{1-\rho}e^{\frac{1-\rho}{\rho}\frac{1}{2}\sum\frac{(\hat{\mu}_i - rp)^2}{\sigma_{\delta i}^2}}$$

where $\hat{\mu}_i$ and $\sigma_{\delta i}$ are the expected mean and standard deviation of the payoff process z, given the investor's prior, private signal, and the price signal.

We compute the expectation E(U) as in Brunnermeier (2001). Some new notation is needed for that. First, denote the excess return as

$$R_i \equiv \hat{\mu}_i - rp_i$$

with mean $\hat{R}_i$. Denote the period zero volatility of $R_i - \hat{R}_i$ as $\hat{V}_i$ (which is just the volatility of R_i). Then, we can write (in a matrix form):

$$U = \frac{1}{1-\rho} W^{1-\rho} r^{1-\rho} e^{\frac{1-\rho}{\rho}\frac{1}{2}[(R-\hat{R})\Sigma_\delta^{-1}(R-\hat{R})+2\hat{R}\Sigma_\delta^{-1}(R-\hat{R})+\hat{R}\Sigma_\delta^{-1}\hat{R}]}$$

Which gives

$$EU = \frac{1}{1-\rho} W^{1-\rho} r^{1-\rho} |I - 2\hat{V}\frac{1-\rho}{2\rho}\Sigma_\delta^{-1}|^{-1/2} \times$$

$$\exp(\frac{(1-\rho)^2}{2\rho^2}\hat{R}\Sigma_\delta^{-1}(I - 2\hat{V}\frac{1-\rho}{2\rho}\Sigma_\delta^{-1})^{-1}\hat{V}\hat{R}\Sigma_\delta^{-1} + \frac{1-\rho}{2\rho}\hat{R}\Sigma_\delta^{-1}\hat{R})$$

and

$$EU = \frac{1}{1-\rho} W^{1-\rho} r^{1-\rho} (\Pi_i(1 - \hat{V}_i\frac{1-\rho}{\rho}\sigma_{\delta i}^{-1}))^{-1/2} \times \exp\left(\frac{1-\rho}{2\rho}\sum \frac{\hat{R}_i^2}{\sigma_{\delta i}}\left[(1 + \frac{\hat{V}_i}{\sigma_{\delta i}}\frac{\rho-1}{\rho})^{-1}\right]\right).$$

Logging the negative of that and simplifying gives

$$-\log(-EU) = const. + \frac{1}{2}\sum_i \log(1 + \frac{\hat{V}_i}{\sigma_{\delta i}}\frac{\rho-1}{\rho}) + \frac{\rho-1}{2\rho}\sum_i \frac{\hat{R}_i^2}{\sigma_{\delta i} + \hat{V}_i\frac{\rho-1}{\rho}}$$

This objective function is strictly decreasing in $\sigma_{\delta i}$ and convex, which means that agents are going to invest all capacity into learning about one asset. For that asset, $\sigma_{\delta i} = e^{-2K}\sigma_{yi}$, and $\sigma_{\delta i} = \sigma_{yi}$ otherwise.

References

Admati, Anat, 1985, A noisy rational expectations equilibrium for multi-asset securities markets, *Econometrica* 53(3), 629–657.

Brunnermeier, Markus K, 2001, *Asset pricing under asymmetric information: Bubbles, crashes, technical analysis, and herding*. (Oxford University Press).

Bucks, Brian K, Arthur B Kennickell, and Kevin B Moore, 2006, Recent changes in U.S. family finances: Evidence from the 2001 and 2004 Survey of Consumer Finances, *Federal Reserve Bulletin* 92, A1–38.

French, Kenneth, 2008, The cost of active investing, *Journal of Finance* 63 (4), 1537–1573.

Kacperczyk, Marcin, Stijn Van Nieuwerburgh, and Laura Veldkamp, 2015, A rational theory of mutual funds' attention allocation, Working Paper New York University.