Investor Sophistication and Capital Income Inequality[*]

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Abstract

We show that capital income inequality is large and growing fast, accounting for a significant portion of total income inequality. We study its determinants in a general equilibrium portfolio choice model with endogenous information acquisition and heterogeneity across household sophistication and asset riskiness. The main mechanism works through endogenous household participation in assets with different risk. The model implies capital income inequality that increases with aggregate information technology. Quantitatively, it generates a path of capital income inequality that matches the evolution of inequality in the U.S.

The rise in wealth and income inequality worldwide has been one of the most hotly discussed topics in academic and policy circles.[1] Evidence on this topic has been largely empirical and focused on heterogeneity in wages and other labor income. In this paper, we emphasize the role of *capital income*–income generated from participation in financial markets. We present new facts on the economic importance of capital income relative to other sources of income inequality, and propose a micro-founded economic mechanism based on information heterogeneity and technological progress to explain the dynamics of capital income inequality. Our mechanism generates dynamics of financial market participation and patterns of trading activity consistent with the data (Calvet, Campbell, and Sodini (2007), Chien, Cole, and Lustig (2011)).

We establish five robust empirical patterns using the 1989-2013 data from the Survey of Consumer Finances (SCF). For the sample of households participating in financial markets (about 34% of the population), we show that (*i*) capital income inequality is an order of magnitude larger than is labor income inequality, and it has been growing at a faster pace, (*ii*) capital income inequality accounts for a quarter of total income inequality, (*iii*) capital income is an important driver of the dynamics of total income inequality over time, (*iv*) capital income has a major impact on the growth in financial wealth inequality. Finally, for the full sample of households, (*v*) it is the growth in inequality within the participating group rather than the extensive margin of participation in financial markets that drives financial wealth inequality.

We propose a mechanism, grounded in a micro-founded theory, that can account for the growth in capital income inequality, qualitatively and quantitatively. The theory is cast as a noisy rational expectations portfolio choice model with endogenous information acquisition subject to capacity constraints, in the spirit of Van Nieuwerburgh and Veldkamp (2009, 2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2015). We generalize these setups by allowing for a nontrivial heterogeneity

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[1]For a summary, see Piketty and Saez (2003); Atkinson, Piketty, and Saez (2011). A comprehensive discussion is also offered in the 2013 Summer issue of the Journal Economic Perspectives and in Piketty (2014).

across investors and assets. First, all investors are endowed with a heterogeneous but positive amount of capacity for processing information. Hence, everyone in the economy learns about asset payoffs, but to different degrees: *sophisticated* investors have greater capacity to process information than do *unsophisticated* ones. This feature of the model is important to explain how aggregate capacity growth affects inequality through its effects on investment behavior of different investor types. Second, financial assets differ in their fundamental volatilities and thus their relative potential to generate superior returns.

Based on the observed assets characteristics, investors decide which assets to learn about, how much information about them to process, and how much to invest. Both the assets that are being learned about and the mass of investors learning about them are determined endogenously. This endogeneity is crucial for generating a quantitatively meaningful growth in inequality and allows us to test our mechanism against asset-level micro data.

The main implication of our information friction is that in the presence of initial investor heterogeneity, symmetric growth in capacity, interpreted as a general progress in information-processing technologies, disproportionately benefits sophisticated households and leads to a growing capital income inequality. This result reflects two characteristics of learning in equilibrium. First, learning exhibits *preference for volatility*: All else equal, individuals choose to learn about volatile assets. Second, there is *strategic substitutability* in learning: The value of learning diminishes as more individuals learn about a given asset, through a general equilibrium effect on prices. Less sophisticated individuals are more responsive to the general equilibrium price effects because their information rents are lower. As a result, symmetric growth in capacity leads to an expansion of sophisticated ownership across asset classes, starting with the most volatile and continuing to lower volatility assets. Simultaneously, unsophisticated individuals retrench from risky assets and hold safer assets. In terms of the aggregate moments, growth in aggregate capacity leads to lower average market

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returns and higher asset turnover. These results play an important role in that they cut against plausible alternative explanations of the observed growth in inequality, such as models with heterogeneity in risk aversion or trading costs.

We quantify the implications of our information friction on capital income inequality in three steps. We consider three types of investors: institutional, sophisticated retail investors and unsophisticated retail investors. We then parameterize the model using U.S. micro-level data on stocks from CRSP and aggregate retail and institutional portfolios from Thomson Reuters for the period 1989–2000, combined with household-level data from the Survey of Consumer Finances. This allows us to pin down the investment environment, including the asset payoffs and risk aversion, as well as heterogeneity in sophistication between investors. Second, we simulate the model using the time series on institutional ownership to pin down the time series of aggregate information capacity in the model, with growth in capacity affecting all agents in a symmetric way. This gives us a time series for capital income inequality between the two retail investors modeled by us. Third, we use the second half of the micro-level stock market data (from 2001 to 2012) to test the information mechanism in the time series.

In our analysis of capital income inequality, we link households' initial sophistication to their initial total wealth. Intuitively, when information about financial assets is costly to process, individuals with different access to financial resources differ in terms of their access to information about financial investments. We take this point as a guiding principle in mapping households into two different wealth groups in the SCF. Specifically, for households that participate in financial markets, we use the average total wealth of the 10% wealthiest individuals relative to that of the 50% poorest individuals in 1989 as a proxy for *initial* relative sophistication. This view of wealth as proxy for sophistication is supported by empirical micro-evidence literature on the strong connection between measures of portfolio sophistication and wealth

(Calvet, Campbell, and Sodini (2009b), Vissing-Jorgensen (2004)).[2]

The model generates capital income inequality growth of 121%, compared with 111% in the data. The main force behind the inequality is that unsophisticated individuals shift their portfolio allocations towards safe, low-volatility assets. This rebalancing effect is quantitatively the most important force in the model: In a one-asset setting, capital income inequality growth is reduced by more than 80%. It is also consistent with empirical evidence from the U.S. Over time, unsophisticated households increase their share of liquid, money-like instruments and shift away from direct stock ownership and ownership of intermediated products, such as actively managed equity mutual funds.[3]

To test our economic mechanism, we look at micro-level evidence on holdings of retail and active institutional portfolios. First, institutional portfolios on average earn rates of return that are approximately 3 percentage points per year higher (1.7-2.2 percent in the model). Second, institutional portfolios in the data have higher ownership shares of equities first for the most volatile stocks and subsequently for stocks with medium and low volatility. Similar result is predicted by our model when we introduce positive shocks to aggregate information capacity. Third, the data exhibit asset turnover that is increasing over time, and increasing in the cross section of stocks sorted by past volatility, which again is a prediction of our model with aggregate capacity growth. We argue that these micro-level patterns are a specific feature of our information friction and would be hard to reconcile with other frictions or sources of heterogeneity.

Intuitively, three forces in the model guide these results. First, more sophisticated investors adjust, state by state, their portfolio holdings better towards assets with higher realized excess returns because they receive higher-precision signals about payoffs. This force drives the difference in returns and asset turnover across investor

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[2]For a theoretical model showing how such a relationship can arise in equilibrium, see Arrow (1987).

[3]More broadly, this pattern matches evidence of a growing retrenchment of retail investors from trading and stock market ownership in general (Stambaugh (2014)).

types. Second, rents from learning are higher for higher volatility assets. As capacity expands and rents from high volatility assets fall in equilibrium, investors expand learning to lower volatility assets. Third, more sophisticated investors are better at capturing informational rents from the increase in aggregate capacity. Thus, they increase more than proportionally their per-capita share in every risky asset they learn about. This has a general equilibrium effect through prices, which pushes unsophisticated individuals to reduce their exposure to assets with large sophisticated ownership. The second and the third forces are jointly responsible for the pattern of expansion of institutional ownership in response to growth in aggregate capacity, which in turn drives asset turnover in the time series and the cross section of assets.

Related literature   Our paper spans three strands of literature: household finance, rational inattention, and income inequality. While some of our contributions are specific to each individual stream, a unique feature of our work is that we integrate the streams into one unified framework.

Within the household finance literature, we build upon the empirical literature on limited capital market participation, growing institutional ownership, household trading decisions, and investor sophistication[4]. While the majority of the studies attribute limited participation rates to differences in participation costs[5] or preferences, we relate investment decisions to differential access to information across households.

In regard to the endogenous information choice, our work is broadly related to Sims (1998, 2003). More germane to our application are the models of costly information of Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2015), from which we depart by exploring the role of asset and investor heterogeneity both analytically and quantitatively. Allowing

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[4]Studies on participation include Mankiw and Zeldes (1991) and Ameriks and Zeldes (2001). Gompers and Metrick (2001) show trends in ownership; Barber and Odean (2001), Campbell (2006), Calvet, Campbell, and Sodini (2009b, 2009a), Guiso and Sodini (2012) analyze household trading decisions; Barber and Odean (2000, 2009), Calvet, Campbell, and Sodini (2007), Grinblatt, Keloharju, and Linnainmaa (2012) examine investor sophistication.

[5]See Gomes and Michaelides (2005), Favilukis (2013).

for such non-trivial heterogeneity produces very different implications for portfolio decisions, asset prices, and the evolution of inequality over time.

The literature on income inequality dates back to the seminal work by Kuznets (1953).[6] In contrast to our paper, a vast majority of that literature focuses on total income or income earned in labor market,[7] and does not relate inequality to heterogeneity in the informational sophistication of households.

The closest paper in spirit to ours is Arrow (1987), who also considers information differences to explain the income gap. However, he does not consider endogenous information acquisition and is not a general equilibrium analysis of the economy with heterogeneously informed agents and many assets. Both of these elements are crucial for our results, especially to establish the validity of our mechanism. Another related paper is Peress (2004), who examines the role of wealth and decreasing absolute risk aversion in investors' information acquisition and participation in one risky asset. However, his focus is not on capital income inequality. Moreover, we show that heterogeneity across assets and agents is a crucial component to quantitatively capture the evolution of capital income inequality and its underlying economic mechanism.

Section 1 establishes five facts regarding income inequality that motivate our analysis. Section 2 presents the theory. Section 3 derives analytic predictions, which we subsequently take to the data. Section 4 presents the quantitative results about the model parameterization and the evolution of capital income inequality, and tests the mechanism using a set of dynamic predictions. Section 5 concludes. All proofs and derivations are in the Appendix and the Online Appendix.

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[6]It has been subsequently advanced by the work of Piketty (2003), Piketty and Saez (2003), Autor, Katz, and Kearney (2006), Atkinson, Piketty, and Saez (2011), and Alvaredo, Atkinson, Piketty, and Saez (2013)

[7]A notable exception is a recent paper by Pástor and Veronesi (2015) who study theoretically a link between redistributive taxes, entrepreneurship, and income inequality. Their model takes advantage of heterogeneity in skill and risk aversion and is not information based.

1 Motivating Facts

This section presents a set of facts on capital income inequality that motivate our study, based on the data from the *Survey of Consumer Finances* (SCF) from 1989 to 2013. The SCF is a standard testing ground for questions related to household finance and thus a reliable source for our purpose.[8]

Since we seek to understand the role of financial markets in generating growth in inequality, we restrict our sample to households that participate in such markets. We define as *participants* households that report holding stocks, bonds, mutual funds, receiving dividends, or having a brokerage account. On average, 34% of households participate, ranging between 32% in 1989, a 40% high in 2001, and 28% low in 2013.[9]

Capital income is income from dividends, taxable and non-taxable interest income, and realized capital gains.[10] During the period 1989-2013, capital income represents an average 14% of total income for households that participate in financial markets. Labor income (income from wages and salaries) represents 56% of total income, while other income makes up the remaining 30%.[11] In terms of its dynamics, the share of capital income in total income falls in the second wave of the SCF survey, from 19.2% in 1989 to 13.5% in 1992, and remains stable after, reaching a value of 13.7% in 2013.

We measure capital income inequality in the participating group in two steps. We first sort the sample of participants by the level of total wealth. Next, we calculate

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[8]For example, Saez and Zucman (2014) show that SCF trends in household wealth at the very top of the wealth distribution are consistent with those obtained from detailed tax records.

[9]As a robustness check, we also consider a broader measure of participation that includes all households with equity in a retirement account. This inclusion raises the participation rates to 35% in 1989, 44% in 2001, and 37% in 2013. All relevant conclusions remain unchanged.

[10]Our definition of capital income is typical in the literature (see, for example, Bucks et al. (2006), Kennickell (2006), Bucks et al. (2009), Alvaredo et al. (2013). The Online Appendix provides detailed definitions of all variables.

[11]Other income includes social security and other pension income, income from professional practice, business or limited partnerships, income from net rent, royalties, trusts and investment in business, unemployment benefits, child support, alimony and income from welfare assistance programs. In the literature on labor income inequality, business income is sometimes included in labor income. The split between labor and other income does not impact our calculations regarding the relative importance of capital income.

inequality as a ratio of income for the top 10% of participants and the bottom 50%.

Fact 1: Capital Income Inequality is Large and Growing Fast   Panel (a) of Figure 1 shows that in the cross-section, capital income is an order of magnitude more unequal than either labor or total income. For example, in 1989, the capital income of the top 10% of participants was 61 times larger than that of the bottom 50% of participants. This ratio increased to 129 in 2013. By comparison, the corresponding ratio for wage income was 3.3 in 1989 and 5.6 in 2013. To compare the dynamics of inequality across income sources, we normalize the inequality of each income measure to 1 in 1989, and plot growth rates for capital, labor, and total income inequality in Panel (b) of Figure 1. Capital income inequality doubled over the sample period, outpacing the growth in labor income inequality, which increased 1.5 times.

| (a) Raw | (b) Normalized |
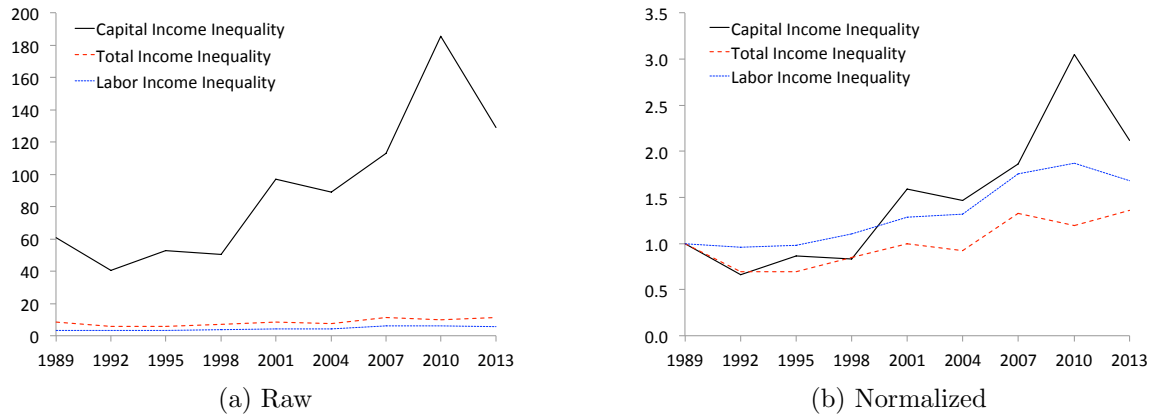
Figure 1: Income inequality growth in the SCF. Inequality is the ratio of the top 10% and the bottom 50% (in terms of total wealth) of participants in financial markets.

Fact 2: Capital Income Inequality Accounts for 25% of Total Income Inequality   To understand the relative importance of various components of total income inequality, in equation (1) below, we decompose total income inequality in period t (denoted T_{10t}/T_{50t}) into shares coming from capital income (denoted by K), labor income (denoted by W), and other (residual) income (denoted by R). This

process integrates two empirical drivers of inequality: the evolution of shares and the evolution of inequality within each income source.

$$\frac{T_{10t}}{T_{50t}} = \frac{K_{10t}}{K_{50t}}\frac{K_{50t}}{T_{50t}} + \frac{W_{10t}}{W_{50t}}\frac{W_{50t}}{T_{50t}} + \frac{R_{10t}}{R_{50t}}\frac{R_{50t}}{T_{50t}} \tag{1}$$

Figure 2 plots the contribution at time t of each of the components of total income to the inequality in total income. On average, 26% of the total income inequality in each year is attributable to capital income. Even though the contribution of capital income to total income inequality has been declining somewhat, from an average of 30% in the first half of the sample (roughly the 1990s) to 23% in the second half of the sample, it still constitutes an economically significant number.

(a) Level   (b) Growth
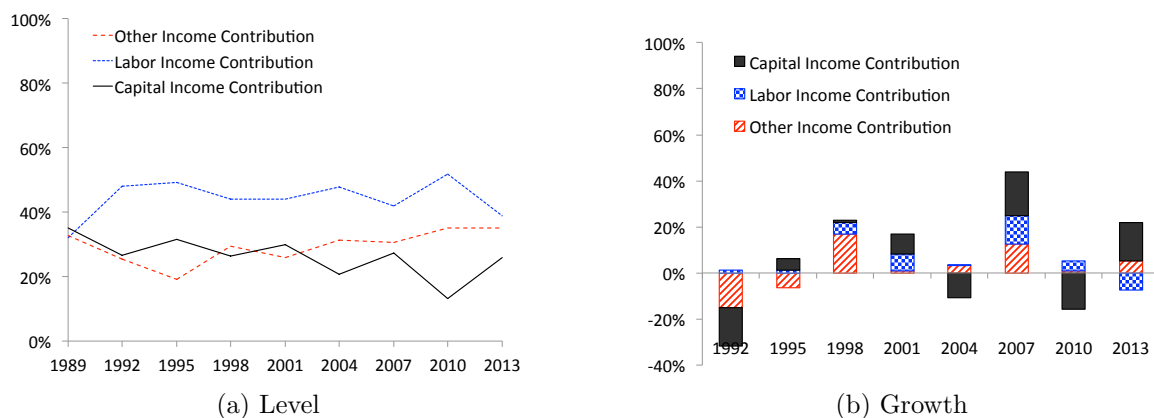
Figure 2: Decomposition of total income inequality in the SCF into contributions from capital income, labor income, and other income.

Fact 3: Capital Income is an Important Driver of Total Inequality Dynamics The dynamics of capital income inequality also significantly contribute to the dynamics of total inequality. Panel (b) of Figure 2 presents a decomposition of the change in total income inequality into changes attributable to capital, labor, and other income. We find that capital income is the biggest contributor to total income inequality–its contribution is on average 2.2 times greater than that of the sum of wage and other income.

Fact 4: Capital Income Drives Financial Wealth Inequality To assess the importance of capital income as a driving force of financial wealth inequality, we generate the counterfactual financial wealth obtained from accruing capital income only.[12] Specifically, starting from 1989, for each wealth decile in the SCF we derive a hypothetical wealth level for subsequent years by accumulating the reported capital income.[13]
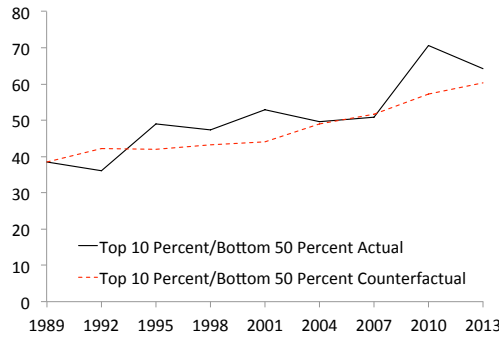

Figure 3: Financial Wealth in the SCF: Actual and counterfactual due to accrual of capital income only.

Figure 3 shows the time series for actual and counterfactual financial wealth inequality. The two series are remarkably close, which suggests an important role for capital income in the evolution of financial wealth inequality.[14] Based on Figure 3 it seems that looking at past capital income realizations may be sufficient to explain the evolution of financial wealth, without resorting to mechanisms that involve savings rates from other income sources. Still, we treat this evidence as suggestive, since our exercise imposes a panel interpretation on a repeated cross-section.

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[12]Financial wealth is the sum of holdings of assets that generate capital income, such as directly held stocks, bonds, and non money market funds, plus the cash value of life insurance, retirement accounts, CDs, other financial assets such as future royalties, annuities, trusts, or managed accounts, plus the value of liquid assets, such as checking and savings accounts, cash, and money accounts.

[13]For example, the counterfactual financial wealth level in 1995 is equal to the actual financial wealth in 1989 plus 3 times the capital income reported in the prior survey years (in this case, 1989 and 1992).

[14]By construction, the two wealth levels are identical in 1989, so the figure also implies that the counterfactual levels of financial wealth for each group are very close to those in the data.

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Fact 5: Participation Does Not Drive Financial Wealth Inequality We study the role that the extensive (participation) and the intensive (investment) margins play in generating growth in financial wealth inequality. Starting in 1989, we plot two financial wealth time series: inequality between non-participants in financial markets and the bottom 50% of the wealth distribution of participants (the extensive margin), and inequality between the bottom 50% and the top 10% of the wealth distribution of participants. Figure 4 plots the two series, normalized to 1 in 1989. As is clear from the figure, there is no significant effect of the participation margin; in turn, inequality has grown substantially for households *within* the participating group.
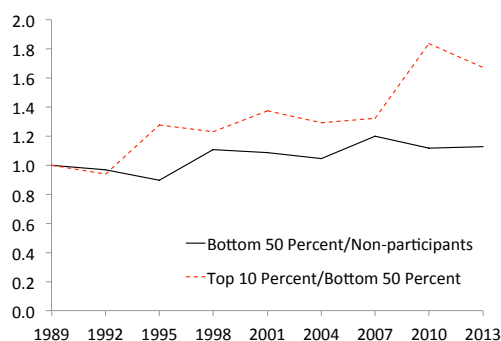
Figure 4: Financial wealth inequality in the SCF: Market participants are sorted in terms of their total wealth.

Overall, our evidence points to capital income being a significant component of total income, and to capital income being an important factor in the growth of both financial wealth and total income inequality. Further analysis of the cross-sectional differences between individuals, discussed in detail in the Online Appendix, shows that higher-wealth individuals, such as the top participants, use more sophisticated investment instruments and invest a lower proportion of their assets in money-like instruments. This suggest access to more sophisticated investment strategies which may generate divergence in capital income over time. Next, we set out to understand the theoretical underpinnings of this evolution of capital income inequality.

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2   Theoretical Framework

A continuum of atomless investors of mass one, indexed by j, solve a sequence of static portfolio choice problems, so as to maximize mean-variance utility over wealth W_j in each period, given common risk aversion coefficient $\rho > 0$. The financial market consists of one risk-free asset, with price normalized to 1 and payoff r, and $n > 1$ risky assets, indexed by i, with prices p_i, and independent payoffs $z_i = \overline{z} + \varepsilon_i$, with $\varepsilon_i \sim \mathcal{N}(0, \sigma_i^2)$. The risk-free asset has unlimited supply, and each risky asset has fixed supply, $\overline{x}$. For each risky asset, non-optimizing "noise traders" trade for reasons orthogonal to prices and payoffs (e.g., liquidity, hedging, or life-cycle reasons), such that the net supply available to the (optimizing) investors is $x_i = \overline{x} + \nu_i$, with $\nu_i \sim \mathcal{N}(0, \sigma_x^2)$, independent of payoffs and across assets.[15]

Prior to making their portfolio decisions investors choose to obtain information about some or all of the risky assets. Mass $\lambda \in (0, 1)$ of investors, labeled *sophisticated*, have high capacity for processing information, K_1, and mass $1 - \lambda$, labeled *unsophisticated*, have low capacity, K_2, with $0 < K_2 < K_1 < \infty$. Information is obtained in the form of endogenously designed signals on asset payoffs subject to this capacity limit. The signal choice is modeled using entropy reduction as a measure of the amount of acquired information (Sims (2003)).

2.1   Investor Optimization

Optimization occurs in two stages. In the first stage, investors solve their information acquisition problem: they choose the distribution of signals to receive in order to maximize expected utility, subject to their information capacity. In the second stage, given the signals they receive, investors update their beliefs about the payoffs and choose their portfolio holdings to maximize utility. We first describe the optimal

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[15]For simplicity, we introduce heterogeneity only in the volatility of payoffs, although the model can easily accommodate heterogeneity in supply and in mean payoffs.

portfolio choice in the second stage, for a given signal choice. We then solve for the ex-ante optimal signal choice.

Portfolio Choice Given equilibrium prices and posterior beliefs, each investor solves

$$U_j = \max_{\{q_{ji}\}_{i=1}^n} E_j(W_j) - \frac{\rho}{2} V_j(W_j) \tag{2}$$

$$s.t. \ \ W_j = r\left(W_{0j} - \sum_{i=1}^n q_{ji} p_i\right) + \sum_{i=1}^n q_{ji} z_i, \tag{3}$$

where E_j and V_j denote the mean and variance conditional on investor j's information set, and W_{0j} is initial wealth. Optimal portfolio holdings are given by

$$q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho \widehat{\sigma}_{ji}^2}, \tag{4}$$

where $\widehat{\mu}_{ji}$ and $\widehat{\sigma}_{ji}^2$ are the mean and variance of investor j's posterior beliefs about payoff z_i.

Information Acquisition Choice Each investor can choose to receive a separate signal s_{ji} on each of the asset payoffs, z_i. Given the optimal portfolio choice, each investor chooses the optimal distribution of signals to maximize the ex-ante expected utility, $E_{0j}[U_j]$. The choice of the vector of signals $s_j = (s_{j1}, ... s_{jn})$ about the vector of payoffs $z = (z_1, ..., z_n)$, is subject to an information capacity constraint, $I(z; s_j) \leq K_j$, where $I(z; s_j)$ denotes the Shannon (1948) mutual information, quantifying the information that the vector of signals conveys about the vector of payoffs. The capacity constraint imposes a limit on the amount of uncertainty reduction that the signals can achieve. Since perfect information requires infinite capacity, each investor faces some residual uncertainty about the realized payoffs.

For tractability, we make the following assumption about the signal structure:

Assumption 1. The signals s_{ji} are independent across assets and investors.

Assumption 1 implies that the total quantity of information obtained by an investor can be expressed as a sum of the quantities of information obtained for each asset.[16] The information constraint becomes $\sum_{i=1}^{n} I(z_i; s_{ji}) \leq K_j$, where $I(z_i; s_{ji})$ measures the information conveyed by the signal s_{ji} about the payoff of asset i.

Investors decompose each payoff into a lower-entropy signal component and a residual component that represents the information lost through this compression: $z_i = s_{ji} + \delta_{ji}$. For tractability, we introduce the following additional assumption:

Assumption 2. For each asset and investor, the signal s_{ji} is independent of the data loss δ_{ji}.

Since z_i is normally distributed, Assumption 2 implies that s_{ji} and δ_{ji} are also normally distributed. By Cramer's Theorem, $s_{ji} \sim \mathcal{N}\left(\bar{z}, \sigma_{sji}^2\right)$ and $\delta_{ji} \sim \mathcal{N}\left(0, \sigma_{\delta ji}^2\right)$ with $\sigma_i^2 = \sigma_{sji}^2 + \sigma_{\delta ji}^2$.[17] Hence, posterior beliefs are normally distributed random variables, independent across assets, with mean $\widehat{\mu}_{ji} = s_{ji}$ and variance $\widehat{\sigma}_{ji}^2 = \sigma_{\delta ji}^2$. Intuitively, a perfectly precise signal results in no information loss, such that posterior uncertainty is zero. Conversely, a signal that consumes no information capacity discards all information about the realized payoff, returning only the mean payoff, $\bar{z}$, and leaving an investor's posterior uncertainty equal to her prior uncertainty.

Using this signal structure and the resulting distribution of expected excess returns, the investor's information problem becomes choosing the variance of posterior beliefs to solve

$$\max_{\{\widehat{\sigma}_{ji}^2\}_{i=1}^n} \sum_{i=1}^{n} G_i \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} \quad s.t. \quad \prod_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} \leq e^{2K_j}, \tag{5}$$

where G_i represents the equilibrium utility gain from learning about asset i.[18] This

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[16]Assumption 1 is common in the literature. Allowing for potentially correlated signals requires a numerical approach, and is beyond the scope of this paper.

[17]In general, the optimal signal structure may require correlation between the signal and the data loss, but Assumption 2 maintains analytical tractability.

[18]The investor's objective omits terms that do not affect the optimization. See Appendix for detailed derivations.

gain is a function of the distribution of expected excess returns only, and hence it is common across investor types and taken as given by each investor.

Lemma 1. *The solution to the maximization problem (5) is a corner: each investor allocates her entire capacity to learning about a single asset from the set of assets with maximal utility gains. The posterior beliefs of an investor j learning about asset $l_j \in \arg\max_i G_i$, are normally distributed, with mean and variance given by*

$$\widehat{\mu}_{ji} = \begin{cases} s_{ji} & \text{if } i = l_j \\ \overline{z} & \text{if } i \neq l_j \end{cases} \quad \text{and} \quad \widehat{\sigma}_{ji}^2 = \begin{cases} e^{-2K_j}\sigma_i^2 & \text{if } i = l_j \\ \sigma_i^2 & \text{if } i \neq l_j. \end{cases} \quad (6)$$

Conditional on the realized payoff z_i, the signal is normally distributed with mean $E\left(s_{ji}|z_i\right) = \overline{z} + \left(1 - e^{-2K_j}\right)\varepsilon_i$, and variance $V\left(s_{ji}|z_i\right) = \left(1 - e^{-2K_j}\right)e^{-2K_j}\sigma_i^2$.

The linear objective function and the convex constraint imply that each investor specializes, learning about a single asset. She always picks an asset with the highest gain G_i and hence all assets that are learned about in equilibrium will have the same gains. Which assets these are is determined in equilibrium.

2.2 Equilibrium

Equilibrium Prices Given the solution to each investor's portfolio and information problem, equilibrium prices are linear combinations of the shocks.

Lemma 2. *The price of asset i is given by $p_i = a_i + b_i\varepsilon_i - c_i\nu_i$, with*

$$a_i = \frac{1}{r}\left[\overline{z} - \frac{\rho\sigma_i^2\overline{x}}{(1 + \Phi_i)}\right], \quad b_i = \frac{\Phi_i}{r(1 + \Phi_i)}, \quad c_i = \frac{\rho\sigma_i^2}{r(1 + \Phi_i)}, \quad (7)$$

where $\Phi_i \equiv m_{1i}\left(e^{2K_1} - 1\right) + m_{2i}\left(e^{2K_2} - 1\right)$ measures the information capacity allocated to learning about asset i in equilibrium, $m_{1i} \in [0, \lambda]$ is the mass of sophisticated investors who choose to learn about asset i, and $m_{2i} \in [0, 1 - \lambda]$ is the mass of un-

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sophisticated investors who choose to learn about asset i, with $\sum_{i=1}^{n} m_{1i} = \lambda$ and $\sum_{i=1}^{n} m_{2i} = 1 - \lambda$.

The price of an asset reflects the asset's payoff and the supply shocks, with relative importance determined by the mass of investors learning about the asset. If there is no information capacity in the economy ($K_1 = K_2 = 0$), or for assets that are not learned about ($m_{1i} = m_{2i} = 0$), the price only reflects the supply shock ν_i. As the capacity allocated to an asset increases, the asset's price co-moves more strongly with the underlying payoff (c_i decreases and b_i increases, though at a decreasing rate). In the limit, as $K_j \to \infty$, the price approaches the discounted realized payoff, z_i/r, and the supply shock becomes irrelevant for price determination.

Equilibrium Learning   Using equilibrium prices, we determine the assets that are learned about and the mass of investors learning about each asset. Without loss of generality, let assets be ordered such that $\sigma_i > \sigma_{i+1}$ for all $i \in \{1, ..., n-1\}$. Let $\xi_i \equiv \sigma_i^2 (\sigma_x^2 + \bar{x}^2)$ summarize the properties of asset i. Then the gain from learning about asset i becomes[19]

$$G_i = \frac{1 + \rho^2 \xi_i}{(1 + \Phi_i)^2}. \tag{8}$$

Lemma 3. *The allocation of information capacity across assets, $\{\Phi_i\}_{i=1}^{n}$, is uniquely pinned down by equating the gains from learning among all assets that are learned about, and by ensuring that all assets not learned about have strictly lower gains:*

$$G_i = \max_{h \in \{1,...,n\}} G_h, \qquad \forall i \in \{1, ..., k\}, \tag{9}$$

$$G_i < \max_{h \in \{1,...,n\}} G_h, \qquad \forall i \in \{k+1, ..., n\}, \tag{10}$$

where k denotes the endogenous number of assets with strictly positive learning mass.

Let m_i denote the total mass of investors learning about asset i and let $c_{i1} \equiv \sqrt{\frac{1+\rho^2 \xi_i}{1+\rho^2 \xi_1}} \leq 1$ denote the exogenous value of learning about asset i relative to asset

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[19]See Appendix for derivation.

1 *(excluding strategic substitutability effects). In a symmetric equilibrium in which* $m_{1i} = \lambda m_i$ *and* $m_{2i} = (1 - \lambda) m_i$, *the masses* $\{m_i\}_{i=1}^n$ *are given by*

$$m_i = \frac{c_{i1}}{C_k} + \frac{1}{\phi}\left(\frac{kc_{i1}}{C_k} - 1\right), \qquad \forall i \in \{1, ..., k\}, \tag{11}$$

$$m_i = 0, \qquad \forall i \in \{k+1, ..., n\}, \tag{12}$$

where $C_k \equiv \sum_{i=1}^k c_{i1}$, *and* $\phi \equiv \lambda\left(e^{2K_1} - 1\right) + (1 - \lambda)\left(e^{2K_2} - 1\right)$ *is a measure of the total capacity for processing information available in the economy, with* $\Phi_i = \phi m_i$.

The model uniquely pins down the total capacity allocated to each asset, Φ_i, but it does not separately pin down m_{1i} and m_{2i}. Since the asset-specific gain from learning is the same for both types of investors, we assume that the participation of sophisticated and unsophisticated investors in learning about each asset is proportional to their mass in the population. In turn, this implies a unique set of masses $\{m_i\}_{i=1}^n$.

Learning in the model exhibits preference for volatility (high σ_i^2) and strategic substitutability (low m_i). Furthermore, the value of learning about an asset also falls with the aggregate amount of information in the market (ϕ), since higher capacity overall increases the co-movement between prices and payoffs, thereby reducing expected excess returns:

$$\frac{\partial G_i}{\partial \sigma_i^2} > 0, \quad \frac{\partial G_i}{\partial m_i} < 0, \quad \frac{\partial G_i}{\partial \phi} < 0.$$

For sufficiently low information capacity, all investors learn about the same asset. Since the gains from learning are increasing in volatility, the asset learned about is the most volatile asset: for $\phi \in (0, \phi_1]$, $m_1 = 1$ and $m_i = 0$ for all $i > 1$, where

$$\phi_1 \equiv \sqrt{\frac{1 + \rho^2 \xi_1}{1 + \rho^2 \xi_2}} - 1. \tag{13}$$

This threshold endogenizes single-asset learning as an optimal outcome for low enough

information capacity relative to asset dispersion. As the overall capacity in the economy increases above this threshold, strategic substitutability in learning pushes some investors to start learning about less volatile assets. For sufficiently high information capacity, or alternatively, for low enough dispersion in assets volatilities, all assets are actively traded, thus endogenizing the assumption employed in models with exogenous signals.

We define the thresholds for learning as follows:

Definition 1. Let ϕ_k be such that for any $\phi \leq \phi_k$, at most the first k assets are actively traded (learned about) in equilibrium, while for $\phi > \phi_k$, at least the first $k + 1$ assets are actively traded in equilibrium.

Lemma 3 implies that the threshold values of aggregate information capacity are monotonic: $0 < \phi_1 < \phi_2 < ... < \phi_{n-1}$. Figure 5 shows how growth in the economy's information capacity changes the mass of investors learning about each asset across the volatility spectrum by changing the equilibrium gains from learning. In the presence of assets heterogeneity, even if many assets are learned about, there is heterogeneity in the information capacity allocated to each of the actively traded assets. Since the equilibrium gain is increasing in volatility and decreasing in m_i, the mass of investors learning about each asset is increasing in volatility. In turn, this heterogeneity has implications for holdings, returns, and turnover in the cross-section of assets.[20]

We further characterize learning in response to variation in investor capacities.

Lemma 4. *Let $\phi \in (\phi_{k-1}, \phi_k]$ such that $k > 1$ assets are actively traded. Consider an increase in ϕ such that $k' \geq k$ is the new number of actively traded assets.*

(i) There exists a threshold asset $\bar{\imath} < k'$, such that m_i is strictly decreasing in ϕ for all $i \in \{1, ..., \bar{\imath} - 1\}$ and strictly increasing in ϕ for all $i \in \{\bar{\imath} + 1, ..., k'\}$.

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[20]One could also let the degree of dispersion in asset payoff volatilities vary, which will imply that learning also varies, with periods with high dispersion being characterized by more concentrated learning, and periods with low dispersion characterized by more diversified learning (and hence portfolios).
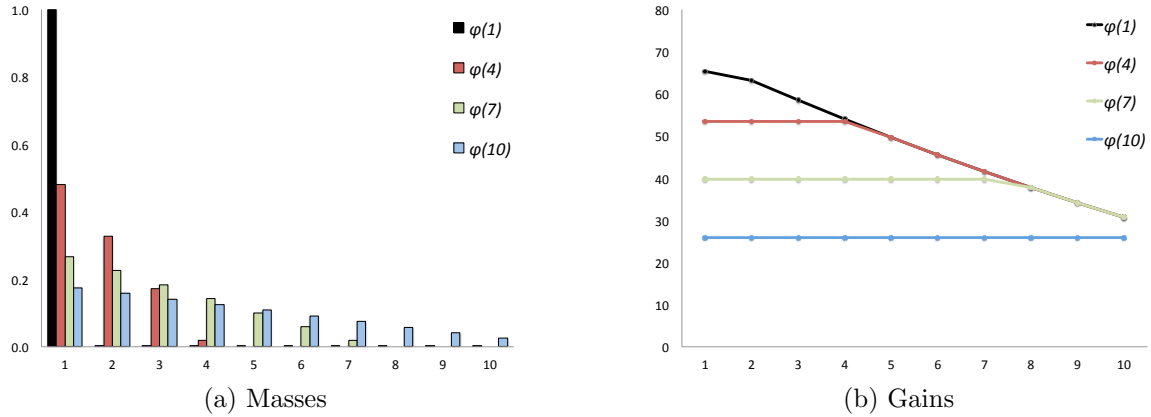
(a) Masses

(b) Gains

Figure 5: The evolution of masses and gains from learning as aggregate capacity is increased. $\phi(k)$ indicates the level of aggregate capacity for which k assets are learned about in equilibrium. On the x-axis, assets are ordered from most (1) to least (10) volatile.

(ii) The quantity (ϕm_i) is increasing in ϕ for all assets $i \in \{1, ..., k'\}$.

(iii) For an increase in ϕ generated by a symmetric growth, $K'_j = (1 + \gamma) K_j$, with $\gamma \in (0, 1)$, the quantity $m_i(e^{2K_j} - 1)$, $j \in \{1, 2\}$, is increasing in K_j at an increasing rate, for $i \in \{\bar{\imath} + 1, ..., k'\}$. For $i \in \{1, ..., \bar{\imath}\}$, $m_i \left(e^{2K_1} - 1 \right)$ grows while $m_i \left(e^{2K_2} - 1 \right)$ grows by less, or even falls if capacity dispersion is large enough.

Lemma 4 shows the diversification effect. First, as the amount of aggregate capacity increases, the mass of investors learning about the most volatile assets decreases, as some investors shift to learning about less volatile assets. Nevertheless, the total amount of capacity allocated to each asset (ϕm_i) strictly increases, to ensure that gains continue to be equated for all assets that are actively traded. This increase means that investment in all assets, including those that have now become less popular, is based on more informative signals. Finally, the increase in aggregate capacity benefits the sophisticated group disproportionately: this group allocates more capacity to each asset relative to the unsophisticated group, which in turn generates asymmetry in investment patterns. In Section 3, we use these results to derive analytic

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predictions on the patterns of investment in response to changes in capacity.

3   Model Predictions

Heterogeneous Capacity   Our first set of analytic results identify the channels through which heterogeneity in information capacity drives capital income inequality in the cross-section, by generating differences in portfolio sizes and compositions both on average and state by state.

Let q_{1i} and q_{2i} denote the average per-capita holdings of asset i for sophisticated and unsophisticated investors, respectively,

$$q_{1i} = \left(\frac{\overline{z}_i - rp_i}{\rho\sigma_i^2} \right) + m_i \left(e^{2K_1} - 1 \right) \left(\frac{z_i - rp_i}{\rho\sigma_i^2} \right), \tag{14}$$

with q_{2i} defined analogously. Per-capita holdings are given by the quantity that would be held under the investors' prior beliefs, with no additional information about the realized payoffs, plus a quantity that is increasing in the realized excess return. The weight on the realized excess return is asset and investor specific, and it captures the amount of information capacity allocated to this asset by this investor group. For actively traded assets, heterogeneity in capacities generates differences in ownership across investor types at the asset level:

$$q_{1i} - q_{2i} = m_i \left(e^{2K_1} - e^{2K_2} \right) \left(\frac{z_i - rp_i}{\rho\sigma_i^2} \right). \tag{15}$$

Integrating over the realizations of the state (z_i, x_i), the expected per-capita ownership difference, as a share of the supply of each asset, is also asset specific,

$$\frac{E\left[q_{1i} - q_{2i} \right]}{\overline{x}} = \left(e^{2K_1} - e^{2K_2} \right) \frac{m_i}{1 + \phi m_i}. \tag{16}$$

Hence, the portfolio of the sophisticated investor is not simply a scaled up version of

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the unsophisticated portfolio. Rather, the portfolio weights *within* the class of risky assets also differ across the two investor types.

Proposition 1 (Ownership). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Then, for $i \in \{1, ..., k\}$,*

(i) $E[q_{1i} - q_{2i}] / \overline{x} > 0$;

(ii) $E[q_{1i} - q_{2i}] / \overline{x}$ is increasing in $E[z_i - rp_i]$;

(iii) $q_{1i} - q_{2i}$ is increasing in $z_i - rp_i$.

The average sophisticated investor (i) holds a larger portfolio of risky assets on average, (ii) tilts her portfolio towards assets with higher expected excess returns, and (iii) adjusts ownership, state by state, towards assets with higher *realized* excess returns. These results identify the channels through which sophisticated investors generate relatively higher capital income, asset by asset, both on average and state by state.

Next, let π_{1i} and π_{2i} denote the capital income per capita from trading asset i, for sophisticated and unsophisticated investors, respectively, with $\pi_{1i} \equiv q_{1i}(z_i - rp_i)$ and $\pi_{2i} \equiv q_{2i}(z_i - rp_i)$. For actively traded assets, heterogeneity in ownership generates heterogeneity in capital income across investor types at the asset level:

$$\pi_{1i} - \pi_{2i} = m_i \left(e^{2K_1} - e^{2K_2}\right) \frac{(z_i - rp_i)^2}{\rho \sigma_i^2}. \tag{17}$$

Integrating over the realizations of (z_i, x_i), the expected capital income difference is

$$E[\pi_{1i} - \pi_{2i}] = \frac{1}{\rho} m_i \left(e^{2K_1} - e^{2K_2}\right) G_i, \tag{18}$$

where G_i is the gain from learning about asset i.

Proposition 2 (Capital Income). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Then, for $i \in \{1, ..., k\}$,*

(i) $\pi_{1i} - \pi_{2i} \geq 0$, with strict inequality in states with non-zero realized excess returns;

(ii) $E[\pi_{1i} - \pi_{2i}]$ *is increasing in asset volatility* σ_i.

The average sophisticated investor realizes larger profits in states with positive excess returns, and incurs smaller losses in states with negative excess returns, because her holdings co-move more strongly with the realized state. Moreover, the biggest difference in profits, on average, comes from investment in the more volatile, higher expected excess return assets.

Larger Capacity Dispersion Our second set of analytic results show that increased dispersion in capacities implies further polarization in holdings, which in turn leads to a growing capital income polarization. Intuitively, greater dispersion in information capacity implies that sophisticated investors receive relatively higher-quality signals about the fundamental payoffs, which enables them to respond more strongly to realized state.

Proposition 3 (**Capacity Dispersion**). *Let* $K_1 > K_2$ *and* $\phi_{k-1} \leq \phi < \phi_k$, *such that the first* $k > 1$ *assets are actively traded in equilibrium. Consider an increase in capacity dispersion of the form* $K_1' = K_1 + \Delta_1 > K_1$, $K_2' = K_2 - \Delta_2 < K_2$, *with* Δ_1 *and* Δ_2 *chosen such that the total information capacity* ϕ *remains unchanged. Then, for* $i \in \{1, ..., k\}$,
(i) Asset prices and excess returns remain unchanged.
(ii) The difference in ownership shares $(q_{1i} - q_{2i})/\overline{x}$ *increases.*
(iii) Capital income gets more polarized as π_{1i}/π_{2i} *increases state by state.*

Increasing the level of capacity dispersion while leaving the aggregate measure of information in the economy unchanged, does not affect equilibrium prices, since keeping ϕ unchanged implies that both the number of assets learned about and the mass of investors learning about each asset remain unchanged. Hence the adjustment reflects a pure transfer of ownership from the relatively unsophisticated investors (who now have even lower capacity) to the more sophisticated investors (who now

have even higher capacity). This reallocation of holdings leads to higher capital income inequality without any general equilibrium effects.

Symmetric Capacity Growth   Our third and most important set of analytic results shows that in the presence of initial heterogeneity, technological progress in the form of *symmetric* growth in information capacity leads to a disproportionate increase in ownership of risky assets by sophisticated investors, and to a growing capital income polarization.

Proposition 4 (Symmetric Growth). *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Consider an increase in ϕ generated by a symmetric growth in capacities to $K_1' = (1 + \gamma) K_1$ and $K_2' = (1 + \gamma) K_2$, $\gamma \in (0, 1)$. Let $k' \geq k$ denote the new equilibrium number of actively traded assets. Then, for $i \in \{1, ..., k'\}$,*

(i) Average asset prices increase and average excess returns decrease.

(ii) Average ownership share of sophisticated investors $E[q_{1i}]/\overline{x}$ increases and average ownership share of unsophisticated investors $E[q_{2i}]/\overline{x}$ decreases.

(iii) Average capital income gets more polarized, as $E[\pi_{1i}]/E[\pi_{2i}]$ increases.

First, higher capacity for processing information means that investors receive more accurate signals about the realized payoffs. Hence, their demand for assets co-moves more closely with the realized state, which implies that prices contain a larger amount of information about the fundamental shocks. As a result, the equilibrium implies lower average returns, larger and more volatile positions, and higher market turnover.

Second, a symmetric growth in capacity that benefits both sophisticated and unsophisticated investors has two effects on portfolio holdings and capital income inequality: a partial equilibrium effect and a general equilibrium effect. Absent any equilibrium price adjustment, the average holdings of risky assets and the comovement between holdings and the realized state increase for both investor types. However, because growth in capacity benefits investors who already have relatively high

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capacity, the benefits accrue more for sophisticated investors. Further, in contrast to the case of increased dispersion, a symmetric change in information capacity affects equilibrium prices. As sophisticated investors increase their demand for risky assets, this drives up average prices, reducing the expected profits of unsophisticated investors, who in turn reduce their average holdings of risky securities.

Trading Volume The differential adjustment to shocks of the two investor types also implies differences in trading intensity, which provides an additional set of testable implications. We divide the investors into 3 groups: (i) sophisticated investors who learn about asset i, with per capita average volume $\overline{V}_i^{SL}$; (ii) unsophisticated investors who learn about asset i, with per capita average volume $\overline{V}_i^{UL}$; and (iii) investors who do not learn about asset i, with per capita average volume $\overline{V}_i^{NL}$. For assets that are not learned about volume is denoted by $\overline{V}_i^{ZL}$. Hence, the total volume generated by the optimizing investors at the asset level is[21]

$$
V_i = \begin{cases} \lambda m_i \overline{V}_i^{SL} + (1-\lambda)\, m_i \overline{V}_i^{UL} + (1-m_i)\, \overline{V}_i^{NL} & \texttt{if } i \text{ is learned about} \\[2mm] \overline{V}_i^{ZL} & \texttt{if } i \text{ is not learned about.} \end{cases}
\tag{19}
$$

We derive an analytic expression for the average per capita volume across states for each asset and investor group, given by

$$
\overline{V}_i^g = \frac{1}{\sqrt{\pi}} \left(\sigma_{qi}^g + \sqrt{\left(\sigma_{qi}^g\right)^2 + \left(\sigma_{\mu i}^g\right)^2} \right),
\tag{20}
$$

where σ_{qi}^g is the cross-sectional standard deviation of holdings across investors in group g and $\sigma_{\mu i}^g$ is the variability of that group's mean holdings across states. Intuitively, trading volume is higher the more disagreement there is in the cross-section of investors and the more the group responds to shocks over time.

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[21]The average volume of the noise traders is exogenous, given by the standard deviation of the noise shock. Among optimizing investors, we assume that investors do not change groups over time. When we take the volume predictions to the data, we compute turnover, which is given by $T_i \equiv V_i/\overline{x}$.

In turn, the degree of cross-sectional disagreement depends on how much capacity investors allocate to learning about that asset, with

$$
\left(\sigma_{qi}^g\right)^2 = \begin{cases} \frac{e^{2K_g}-1}{\rho^2\sigma_i^2} & \text{if } i \text{ is learned about \& g = SL,UL} \\ 0 & \text{if } i \text{ is learned about \& g = NL} \\ 0 & \text{if } i \text{ is not learned about.} \end{cases}
$$

while the degree to which investors adjust holdings over time depends on how much learning is allocated to the asset, both by the particular investor group and by the market overall:

$$
\left(\sigma_{\mu i}^g\right)^2 = \begin{cases} \left(\frac{e^{2K_g}}{1+\phi m_i}\right)^2 \sigma_x^2 + \left(\frac{e^{2K_g}-1-\phi m_i}{1+\phi m_i}\right)^2 \frac{1}{\rho^2\sigma_i^2} & \text{if } i \text{ is learned about \& g = SL,UL} \\ \left(\frac{1}{1+\phi m_i}\right)^2 \sigma_x^2 + \left(\frac{\phi m_i}{1+\phi m_i}\right)^2 \frac{1}{\rho^2\sigma_i^2} & \text{if } i \text{ is learned about \& g = NL} \\ \sigma_x^2 & \text{if } i \text{ is not learned about.} \end{cases}
$$

These expressions enable us to derive a set of testable implications summarized below.

Proposition 5 (Volume). *Let $K_1 > K_2$ and $\phi_{k-1} \le \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Then for assets that are learned about, $i \in \{1, ..., k\}$, average volume is increasing in investor sophistication and is higher for investors who actively trade the asset: $\overline{V}_i^{SL} > \overline{V}_i^{UL} > \overline{V}_i^{NL}$.*

Hence, sophisticated investors generate more asset turnover, since having higher capacity to process information enables them to take larger and more volatile positions, relative to unsophisticated investors. Moreover, assets that are actively traded, in turn, have a higher volume compared with assets that are passively traded (based only on prior beliefs).

Learning from Prices In our analysis so far, we have presented the information acquisition problem in terms of a constraint on information obtained through private

signals alone, abstracting from the possibility of learning from the price realizations. In the Online Appendix, we provide a formal proof that this is in fact an optimal strategy in a world in which learning from prices is an option. Intuitively, if the information contained in prices is costly to process–just like information contained in the private signals–then prices are an inferior source of information compared with the private signals that are designed to provide information specifically about payoffs, since prices are contaminated with information about the noise trader shocks, which are not payoff-relevant per se.

Remark on Risk Aversion Heterogeneity   Capital income inequality can be also driven by differences in investors' risk aversion, in the absence of any heterogeneity in the capacity to process information about asset payoffs. In particular, if one group of investors were less risk averse they would hold a greater share of risky assets, and hence they would have higher expected capital income.[22]   Within our mean-variance specification, a growing difference in risk aversion produces growing *aggregate* ownership in risky assets of less risk averse investors, and a uniform, proportional retrenchment from risky assets of more risk averse investors. However, it does not generate *(i)* differences in portfolio weights within a class of risky assets, *(ii)* investor-specific rates of return on equity, or *(iii)* differential growth in ownership by asset volatility.[23]

4   Quantitative Results

In this section, we parameterize our model using stock-level micro data by asset class and investor type, combined with household-level data from the SCF. The use of stock-level data allows us to parameterize the details of the stochastic environment

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[22]Such setting would also encompass situations in which investors are exposed to different levels of volatility in areas outside capital markets, like labor income.

[23]In a CRRA model, portfolio weights would also be identical *across risky assets*; hence, even in that specification, rates of return on equity would be equalized across investor types.

that the investors face, including asset heterogeneity. Using the SCF allows us to pin down differences in sophistication among the households and to map the model's predictions about capital income inequality to household-level data. We show that the parameterized model generates a path for capital income inequality that is quantitatively close to the data. The critical forces here are *symmetric* aggregate technological progress combined with *initial* heterogeneity.

4.1 Parameterization

The objective of our parameterization is to provide quantitative evidence on capital income inequality among the U.S. households. This requires specifying two types of households: sophisticated and unsophisticated. However, because these investors live in the market economy we need to include the third type of investor, institutional investor. Hence, our parameterization includes three types of investors: institutional investors, sophisticated retail investors, and unsophisticated retail investors.

We use two data sets in our parameterization. The first data set is institutional portfolio holdings database from Thomson Reuters, which contains a large sample of portfolios of publicly traded equity[24] held by institutional investors, and comes from quarterly reports required by law and submitted by institutional investors to the Securities and Exchange Commission.[25] These data help us pin down the fundamental shocks to asset payoffs and noise trader demand, as well as the ratio of capacity of institutional to retail investors. The second data set is the Survey of Consumer Finances, which allows us to map the two retail investor groups into household groups in the SCF. We use the SCF to identify the ratio of capacities between the two retail investor groups, and to compare the model's predictions on capital income inequality

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[24]In our parameterization, we use direct stock holdings and returns, which allows us to use micro-level holdings data. However, we view our results as applying more broadly to other asset classes, such as mutual funds. For a detailed discussion, see Section 4.3.

[25]We identify institutional investors as investment companies or independent advisors (types 3 and 4) in the Thomson data. These investors include wealthy individuals, mutual funds, and hedge funds. Among all types, these groups are particularly active in their information production efforts. Retail investors, in turn, are other investors who are not in the Thomson data.

between the two household groups to the data on capital income inequality in the SCF.

Measure of household sophistication An important element of our analysis is the measurement of investor sophistication. Following the work of Arrow (1987), Calvet et al. (2009a) and Vissing-Jorgensen (2004), we use initial wealth levels as proxies for initial sophistication. We assume that wealthier individuals have access to better information production or processing technologies, i.e. they have greater information capacity. Specifically, for each survey year, we consider two groups of participating households: those who are in the top decile of total wealth (sophisticated retail investors) and those who are in the bottom 50% of total wealth (unsophisticated retail investors). We use the ratio of financial wealth levels in the SCF in 1989 as a target for the ratio of information capacities between sophisticated and unsophisticated retail investors (K_2/K_3).

Empirical Targets The complete list of parameter values and targets is presented in Table 1. For parsimony, we restrict some parameters and normalize the natural candidates. We normalize the mean payoff to $\bar{z}_i = 10$ and asset supply to $\bar{x}_i = 5$ for all assets. We restrict the volatilities of the noise shocks, $\sigma_{xi} = \sigma_x$ for all assets, and set the number of assets $n = 10$. The remaining parameters are the information capacities of the three investor types $(K_1, K_2, \text{ and } K_3)$, the fraction of institutional investors in the population (λ), the fraction of sophisticated investors among the retail investors (δ), the risk-free interest rate (r), the risk aversion parameter (ρ), the volatility of the noise shock (σ_x), and the volatilities of the payoffs, $\{\sigma_i\}_{i=1}^n$, for which we normalize the lowest volatility, $\sigma_n = 1$, and assume that volatility changes linearly across assets.[26]

Motivated by our earlier discussion on the link between total wealth and sophis-

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[26]Specifically, we set $\sigma_i = \sigma_n + \alpha(n-i)/n$, which implies the volatility distribution is parameterized fully by a single parameter α.

Table 1: Parameter Values

Parameter	Symbol	Value	Target (1989-2000 averages)
Mean payoff, supply	$\bar{z}_i, \bar{x}_i$	10, 5 for all i	Normalization
Number of assets	n	10	Normalization
Risk-free rate	r	2.5%	3-month T-bill $-$ inflation $= 2.5\%$
Risk aversion	ρ	1.106	Market return $= 11.9\%$
Vol. of noise shocks	σ_{xi}	0.41 for all i	Average turnover $= 9.7\%$
Vol. of asset payoffs	σ_i	$\in [1, 1.4776]$	p90/p50 of idio return vol $= 3.54$
Information capacities	K_1, K_2	0.4573, 0.4024,	Sophisticated share $= 23\%$
and investor masses	K_3, λ	0.0106, 0.205,	Share actively traded $= 50\%$
	δ	0.16	Institutional return $= 13.4\%$

tication, we set the ratio of capacities of sophisticated versus unsophisticated retail investors to the ratio of the financial wealth of the top 10% versus the bottom 50% of the total wealth distribution in the SCF in 1989, equal to 38. We also pick $\delta = 0.16$, reflecting the 10/50 weights of households we look at in the SCF. The rest of the parameter values is chosen to jointly match key moments from stock-level micro data and aggregate investor-type equity shares for the first half of our sample, 1989-2000. We set the following targets: *(i)* the equity ownership share of sophisticated investors of 23%; *(ii)* the average return on 3-month Treasury bills minus the inflation rate, equal to 2.5%; *(iii)* the average annualized stock market return in excess of the risk-free rate, equal to 11.9%; *(iv)* the average annualized stock market excess return of the institutional investors of 13.4%; *(v)* the average monthly equity turnover (defined as the total monthly volume divided by the number of shares outstanding), equal to 9.7%; *(vi)* the ratio of the 90th percentile to the median of the cross-sectional idiosyncratic volatility of stock returns, equal to 3.54; and *(vii)* the fraction of assets that investors learn about, which, in the absence of empirical guidance, we arbitrarily set to 50%.

As shown in Table 2, the model implies a 2.24 percentage point advantage in the average returns of the institutional portfolio (who earn an average return of 13.4%)

relative to the retail plus noise traders portfolio (who earn an average return of 11.16%) and a 1.7 point advantage relative to only the retail portfolio which does not include noise traders[27]. This difference is comparable to the 3 point difference in the data for the 1989-2000 period (with average portfolio returns of 13.4% versus 10.4%). Thus, the model can generate a significant fraction of the empirical difference in returns (ranging from 57% to 75% of the data), while matching other aggregate targets.

Table 2: Average Portfolio Returns: Data and Model

	1989-2000	
Portfolio Return	Data	Model
Market Return	11.9%	11.6%
Institutional investors	13.4%	13.4%
Retail investors Only	10.4%	11.7 %
Retail investors + noise traders		11.16%

Return Decomposition   As our analytical results suggest, higher sophistication implies better portfolio performance. Specifically, institutional portfolios outperform retail portfolios for two reasons (summarized in Propositions 1 and 2): *(i)* they are more exposed to risk because they hold a larger share of risky assets (compensation for risk); and *(ii)* they have informational advantage (compensation for skill). In order to shed light on the relative importance of these two effects, we decompose the returns of each investor type by computing the unconditional expectation of the return on the portfolio held by investor type $j \in \{I, R\}$:

$$R_j = E \sum_i \omega_{jit}(r_{it} - r) = \sum_i Cov(\omega_{jit}, r_{it}) + \sum_i E\omega_{jit}E[r_{it} - r], \qquad (21)$$

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[27]In addition to the optimizing sophisticated and unsophisticated households, the model features a third type of agent: noise traders, who trade for reasons unrelated to asset payoffs and prices.

where $r_{it} = z_{it}/p_{it}$ is the time t return on asset i and ω_{jit} is the portfolio weight of asset i for investor j at time t as $\omega_{jit} = q_{jit}p_{it}/\sum_l q_{jlt}p_{lt}$. The first term of the decomposition captures the covariance conditional on investor j information set, i.e. the investor's reaction to information flow via portfolio weight adjustment (*skill effect*); the second term captures the *average effect*, unrelated to active trading.

Quantitatively, the skill effect accounts for the majority of the return differential in the model. To show that, we compute the counterfactual return of institutional investors if their skill matched that of retail investors, but their average holdings stayed the same

$$\hat{R}_I = \sum_i Cov(\omega_{Rit}, r_{it}) + \sum_i E\omega_{Iit}E[r_{it} - r]. \tag{22}$$

Such a portfolio would generate an annualized return of 12%, which implies that the compensation for skill accounts for 87% of the 1.7% return differential between the institutional and retail portfolios.

Remark on Delegated Investment The above findings may suggest that households with little information could improve their returns by delegating their portfolios to better-informed actively managed institutions. This process, however, would not come as free because identifying a skilled manager ex ante requires allocating information capacity. For example, extant literature in finance (e.g., Kacperczyk et al. (2005) , Pástor et al. (2015)) finds that while the average abnormal gross returns of mutual funds are positive, the distribution of returns is highly dispersed and the returns are not predictable.[28] Further, even if one could successfully identify such superior investments, delegation is costly which lowers the benefit of switching. Finally, while it is true that the extent of delegated portfolio management has been on the rise for most of our sample period, we show novel results indicating that the

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[28]Empirically, we observe a strong persistence of negative returns and much weaker persistence of positive returns.

last two decades have witnessed a strong asymmetry in investment flows coming from institutional (who enter) vs. retail investors (who leave), especially in the riskiest equity mutual funds. These patterns are consistent with our theoretical predictions and we discuss them in more detail in Section 4.3.[29]

4.2 Dynamics of Capital Income Inequality

We assess our model's quantitative predictions for the evolution of capital income inequality in response to aggregate growth in information technology. Specifically, we simulate the model for 24 years, reflecting the number of years in the SCF, setting the aggregate capacity level in each year to match the observed institutional ownership rate in the stock market data. Hence, the ownership rates are matched both in the model and in the data.[30]
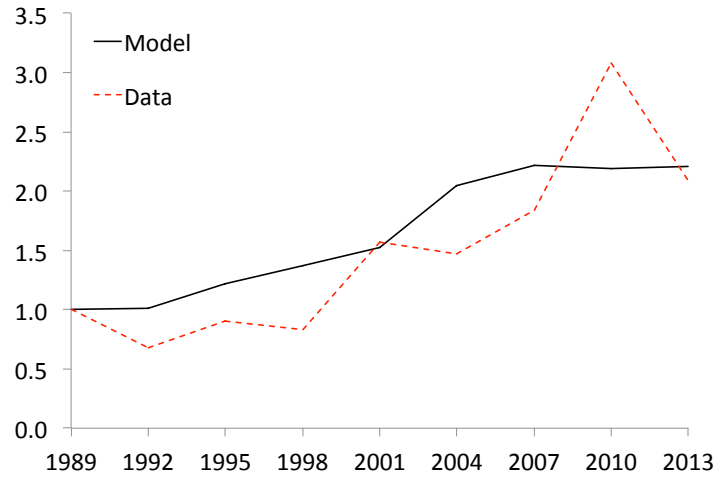

Figure 6: Cumulative Growth in Capital Income Dispersion

The results of this exercise are presented in Figure 6. The model comes very close

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[29]Another alternative for unsophisticated investors would be to participate in the passive market portfolio instead. Although such investment would by design not benefit from information advantage it could benefit from equity risk premium. However, from utility perspective such a portfolio would not be desirable when compared to the information-sensitive investment in low-risk assets, especially taking into consideration positive general equilibrium implications of participation on asset prices.

[30]In the first three years of this exercise, we match the ownership only approximately, as the model implies a lower bound on institutional ownership of 20.5%, higher than the data counterpart for these years (by 2 to 5 percentage points).

to matching the overall growth in inequality in the data, with a 121% growth in the model vs. 111% growth in the data.

The Importance of Heterogeneity   Figure 7 presents results from an alternative specification of the model with only one risky asset. The difference between this specification, labeled *One Asset*, and the benchmark model quantifies the role of asset heterogeneity in driving capital income inequality. [31] The one-asset economy generates growth in capital income inequality that is only 17% of the growth generated by the benchmark model. Hence, asset heterogeneity plays a crucial role in driving capital income inequality in the model. It generates higher payoffs from learning and larger effects on the retrenchment of unsophisticated investors from risky asset markets.

Remark on Constant Relative Risk Aversion Utility   In the Online Appendix, we analyze the model with CRRA utility. Since a closed-form solution to the full model is not feasible, we focus on a local approximation of the utility function. We show that the model solution under no capacity differences predicts the same portfolio shares for risky assets, *independent of wealth.* Intuitively, if agents have common information, then wealth differences affect the composition of their allocation between the risk-free asset and the risky portfolio, but not the composition of the risky portfolio, which is determined optimally by the (common) belief structure. As a result, differences in capacity are a necessary component of the model to generate any risky

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[31]In terms of the parameterization, the model with one asset takes away three targets from the benchmark model: heterogeneity in asset volatility, fraction of actively traded assets and the return of institutional portfolios. We keep the value of risk aversion coefficient the same as in the benchmark model and pick the volatility of the single asset payoff equal to the median payoff volatility of the benchmark model. That leaves four remaining parameters: the fraction λ, overall capacity ϕ, volatility of the noise trader demand σ_x, and the ratio of capacity of institutional versus sophisticated retail investors. We choose these to match: the average market return (11.9%), average asset turnover (9.7%), institutional ownership (23%) and the initial average stock ownership of 15.7% taken from the benchmark model (to make the parameterizations comparable). In simulating the model, we choose aggregate capacity period by period to match institutional ownership, just like in the benchmark model.
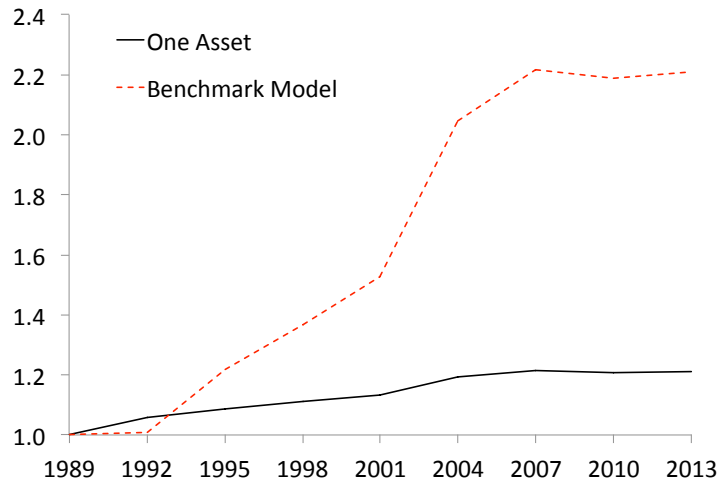
Figure 7: Cumulative Growth in Capital Income Dispersion: The Role of Asset and Capacity Growth Heterogeneity.

return differences across agents.

Remark on Endogenous Capacity Choice   In the benchmark model, we assume an exogenous relation between initial capacity and an investor's wealth. In the Online Appendix, we show how such relation could arise endogenously. Intuitively, if investors endogenously choose different portfolio sizes, then their net benefit of investing in information will increase with portfolio size. We apply this idea in a model in which investors have identical CRRA preferences and make endogenous capacity choice decisions. In the context of the information choice model, CRRA utility specification is not tractable; hence, we map a common relative risk aversion together with wealth differences locally into different absolute risk aversion coefficients. In a numerical example, we show how initial wealth differences observed in the 1989 SCF map into endogenous capacity differences, for different values of the cost of capacity and different relative risk aversion coefficients. We show that for a wide range of the risk aversion specifications and for capacity cost away from zero, the implied differences in capacity are equal or actually larger than the ones specified in the benchmark model. Hence, we view our parameterization as cautious in that it implies modest

initial differences.

4.3 Testing the Mechanism

In this section, we generate a set of dynamic predictions of the model and compare them to the corresponding data moments to provide support for our mechanism. These are robust predictions of our mechanism proven analytically in Section 3. Below, we show a good quantitative fit of these predictions vis a vis the data.

To test our mechanism, we explore the consequences of a symmetric change in capacities of both investor types, targeting the change in the equity ownership share of institutional investors. In the data, this share grew from an average of 23% in the 1989-2000 period to 46% during 2001-2012. We find that the progress in information capacity required to achieve this target amounts to an annual growth of 13% (for 11 years, from the middle of the first sub period to the middle of the second sub period). Hence, in the presence of initial capacity dispersion, subsequent symmetric capacity growth is sufficient to generate a disproportionate growth in sophisticated ownership and retrenchment of unsophisticated investors from risky assets.

Market Averages In the model, symmetric growth in information capacities implies large changes in average market returns, cross-sectional return differentials, and turnover. Table 3 reports the model predictions and their empirical counterparts.

Table 3: Market Averages: Data and Model

	2001-2012	
Statistic	Data	Model
Market Returns	2.4%	1.8%
Institutional portfolio	2.9%	1.83%
Retail portfolio	1.6%	1.77%
Unsophisticated + Noise traders portfolio		1.74%
Average Equity Turnover	16.0%	14.4%
Sophisticated Ownership Share (target)	46.0%	46.0%

Both the model and the data exhibit a decrease in market return and in the return differential between sophisticated and unsophisticated portfolios. The lower market return is a result of an increase in the quantity of information, as prices track payoffs more closely than in the initial sample period, implying lower excess returns. The model also predicts a sharp increase in average asset turnover, in magnitudes consistent with the data. As with the market return, this result is a direct implication of our mechanism and is not driven by changes in fundamental asset volatilities, which remain unchanged. Intuitively, higher turnover is driven by more informed trading by sophisticated investors, due to their holding a larger share of the market and receiving more precise signals about asset payoffs (Proposition 5).

Expansion of Ownership   In our dynamic exercise, we target the overall increase in sophisticated ownership. The expansion occurs in a very specific way across assets, both in the model and in the data. In the model, investors prefer to learn about assets with high volatility, and they initially start learning about the most volatile assets, which increases their holdings of those assets. Further increases in capacity induce them to expand learning to lower-volatility assets, per Lemma 3. In partial equilibrium, this process holds for both investor types. However, in general equilibrium, as institutional investors expand ownership, they take larger positions, which shrinks excess returns. Retail and less sophisticated investors are more responsive to lower excess returns, and retrench.

In Panel (a) of Figure 8, we present the time series of sophisticated ownership across asset volatility classes, generated by the model in response to a sequence of aggregate information capacity underlying the exercise in Section 4.2, which matches institutional ownership year by year. The model predicts that sophisticated investors exhibit the highest initial growth in ownership for the the highest-volatility assets, followed by growth in ownership of the medium-volatility assets, and then growth of the lowest-volatility assets. This prediction is robustly borne out in the data, per

36

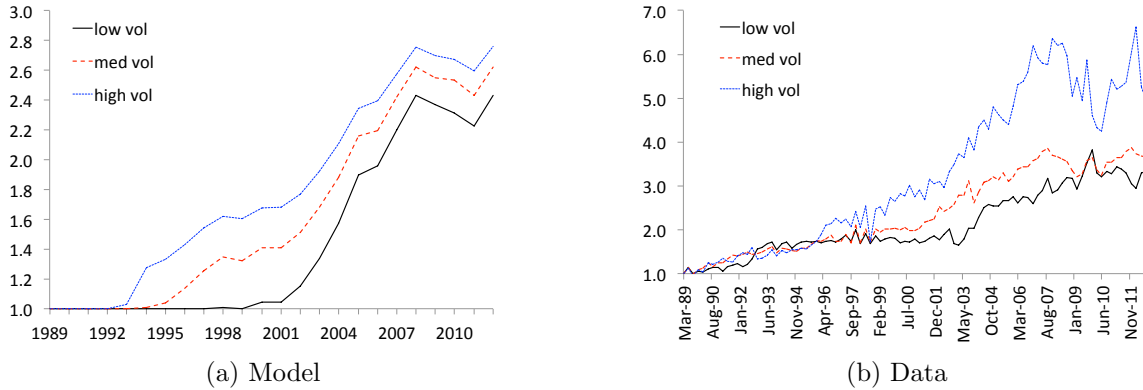(a) Model   (b) Data

Figure 8: Cumulative Growth in Institutional Ownership.

Panel (b) of Figure 8. As we argue in the analytical section, this prediction as unique to our information-based mechanism, hence providing an important model validation.

In conclusion, even though we parameterize the model to match the *aggregate* ownership levels of sophisticated investors in the pre- and post-2000 periods, the model also explains quantitatively *how* ownership changes *across* asset volatility classes, in terms of both the timing of growth levels and the absolute magnitudes of the changes.

Cross-sectional Turnover Our model implies cross-sectional variation in asset turnover. Intuitively, if an asset is more attractive and investors want to trade it, then more investors with precise signals about this asset's returns would want to act on such better information by taking larger and more volatile positions. Since sophisticated investors receive more precise signals, and they have preference for high-volatility assets, we should see a positive relationship between volatility and turnover. Table 4 reports turnover in relation to return volatility in the model and the data.

The first two rows compare data and the model prediction for 1989-2000 subsample. Both data and model show that turnover is increasing in volatility. The results are quantitatively close to each other. In the next two rows, we compare data for the 2001-2012 period to results generated from the dynamic exercise in the model in which we increase overall capacity. The model implies an increase in average

Table 4: Turnover by Asset Volatility

Volatility quintile	1	2	3	4	5	Mean
1989-2000						
Data	5%	8.5%	10.5%	12.5%	11.5%	9.7%
Model	9.25%	9.25%	9.36%	9.8%	10.7%	9.7%
2001-2012						
Data	11%	14.6%	17%	18.4%	19.3%	16%
Model	12.8%	14%	14.8%	15%	15.3%	14.4%

turnover and additionally matches the cross-sectional pattern of this increase. This effect is purely driven by our information friction, since the fundamental volatilities in this exercise remain constant over time.[32]

Retrenchment Across Other Asset Classes We provide auxiliary empirical support in favor of the model's ownership predictions by considering money flows into mutual funds. Equity funds are more risky than non-equity funds; hence, unsophisticated investors should be less likely to invest in the former, especially if aggregate information capacity grows.

We use mutual fund data from Morningstar, which classifies different funds into those serving investors whose investment is at least $100,000 (institutional funds) and those serving investors with investment value less than $100,000 (retail funds). For the purpose of testing our predictions, we define sophisticated investors as those investing in institutional funds and unsophisticated investors as those investing in retail funds. We then calculate cumulative aggregate dollar flows into equity and non-equity funds, separately for each investor type. The data span years 1989-2012.

As shown in Figure 9, the cumulative flows from sophisticated investors into equity and non-equity funds increase steadily over the entire sample period. In contrast, the flows from unsophisticated investors display a markedly different pattern. The

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[32]Our model also implies a positive turnover-ownership relationship, which we confirm in the data. This result is consistent with the empirical findings in Chordia et al. (2011).

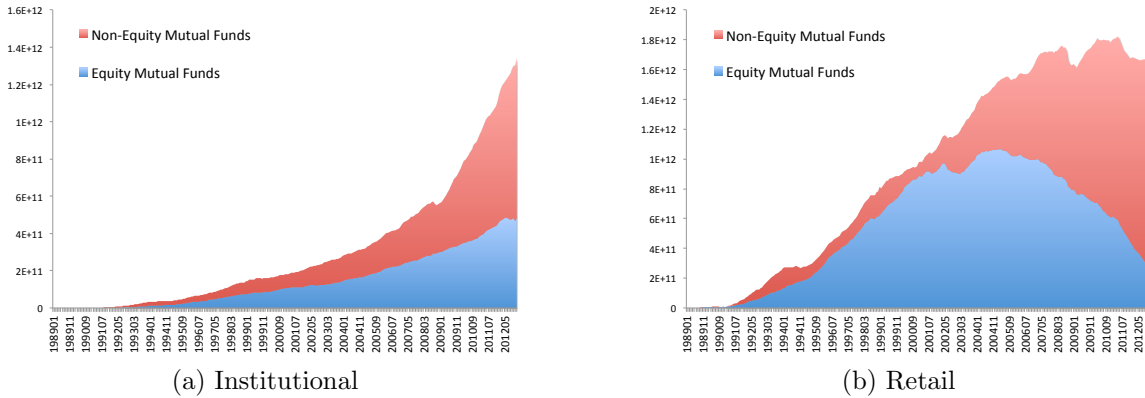| (a) Institutional | (b) Retail |

Figure 9: Cumulative Flows to Mutual Funds: Institutional vs. Retail.

flows into equity funds grow until 2000 but subsequently decrease at a significant rate of more than 3 times by 2012. Moreover, this decrease coincides with a significant increase in cumulative flows to non-equity funds. Notably, the increase in equity fund flows by unsophisticated investors observed in the early sample period is consistent with the steady decrease in holdings of individual equity documented earlier. To the extent that direct equity holdings are more risky than diversified equity portfolios, such as mutual funds, this implies that unsophisticated investors have been systematically reallocating their wealth from riskier to safer asset classes.

Overall, the findings support our model's predictions: Sophisticated households have a large exposure to risky assets and subsequently add exposure to less risky assets, while unsophisticated ones leave riskier assets and increasingly move into safer assets as they face greater information disadvantage.

5 Concluding Remarks

What contributes to the growing income inequality across households? This question has been of great economic and policy relevance for at least several decades starting with the seminal work by Kuznets (1953). We approach this question from the perspective of capital income that is known to be highly unequally distributed

across individuals. We propose a theoretical information-based framework that links capital income derived from financial markets to a level of investor sophistication. Our model implies the presence of income inequality between sophisticated and unsophisticated investors that is growing in the extent of total sophistication in the market, and could be the result of aggregate technological progress. Additional predictions on asset ownership, market returns, and turnover help us pin down the economic mechanism and rule out alternative explanations. The quantitative predictions of the model match qualitatively and quantitatively the observed data.

One could argue that the overall growth of investment resources and competition across investors with different skill levels are generally considered as a positive aspect of a well-functioning financial market. However, our work suggests that one should assess any policy targeting overall information environment in financial markets as potentially exerting an offsetting and negative effect on socially relevant issues, such as distribution of income. Our work also sheds light on the overall benefits and redistribution aspects of progress in financial markets in terms of creating new financial instruments. Depending on where the new assets land on the volatility (or more generally, opaqueness) spectrum, the benefits will accrue to the relatively less (low-volatility assets) or more (high-volatility assets) sophisticated investors.

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Appendix: Proofs

Model

Portfolio Choice. In the second stage, each investor chooses portfolio holdings q_{ji} to solve

$$\max_{\{q_{ji}\}_{i=1}^n} U_j = E_j(W_j) - \frac{\rho}{2} V_j(W_j) \quad s.t. \quad W_j = r\left(W_{0j} - \sum_{i=1}^n q_{ji} p_i\right) + \sum_{i=1}^n q_{ji} z_i,$$

where E_j and V_j denote the mean and variance conditional on investor j's information set:

$$E_j(W_j) = E_j\left[rW_{0j} + \sum_{i=1}^n q_{ji}(z_i - rp_i)\right] = rW_{0j} + \sum_{i=1}^n q_{ji}\left[E_j(z_i) - rp_i\right],$$

$$V_j(W_j) = V_j\left[rW_{0j} + \sum_{i=1}^n q_{ji}(z_i - rp_i)\right] = \sum_{i=1}^n q_{ji}^2 V_j(z_i).$$

Let $\widehat{\mu}_{ji} \equiv E_j[z_i]$ and $\widehat{\sigma}_{ji}^2 \equiv V_j[z_i]$. The investor's portfolio problem is to maximize

$$U_j = rW_{0j} + \sum_{i=1}^n q_{ji}(\widehat{\mu}_{ji} - rp_i) - \frac{\rho}{2}\sum_{i=1}^n q_{ji}^2 \widehat{\sigma}_{ji}^2.$$

The first order conditions with respect to q_{ji} yield $q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho\widehat{\sigma}_{ji}^2}$. Since W_{0j} does not affect the optimization, we normalize it to zero. The indirect utility function becomes

$$U_j = \frac{1}{2\rho}\sum_{i=1}^n \frac{(\widehat{\mu}_{ji} - rp_i)^2}{\widehat{\sigma}_{ji}^2}. \qquad \square$$

Posterior Beliefs. The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that

$$\widehat{\mu}_{ji} = \overline{z} + \frac{Cov(s_{ji}, z_i)}{\sigma_{sji}^2}(s_{ji} - \overline{s}_{ji}) = s_{ji},$$

$$\widehat{\sigma}_{ji}^2 = \sigma_i^2\left(1 - \frac{Cov^2(s_{ji}, z_i)}{\sigma_{sji}^2 \sigma_i^2}\right) = \sigma_{\delta ji}^2. \qquad \square$$

Information Constraint. Let $H(z)$ denote the entropy of z, and let $H(z|s_j)$ denote the conditional entropy of z given the vector of signals s_j. Then

$$I(z; s_j) \equiv H(z) - H(z|s_j) \overset{(1)}{=} \sum_{i=1}^{n} H(z_i) - H(z|s_j) \overset{(2)}{=} \sum_{i=1}^{n} H(z_i) - \sum_{i=1}^{n} H(z_i|z^{i-1}, s_j)$$

$$\overset{(1)}{=} \sum_{i=1}^{n} H(z_i) - \sum_{i=1}^{n} H(z_i|s_j) \overset{(3)}{=} \sum_{i=1}^{n} H(z_i) - \sum_{i=1}^{n} H(z_i|s_{ji}) = \sum_{i=1}^{n} I(z_i; s_{ji})$$

where (1) follows from the independence of the payoffs z_i; (2) follows from the chain rule for entropy, where $z^{i-1} = \{z_1, ..., z_{i-1}\}$; (3) follows from the independence of the signals s_{ji}.

For each asset i, the entropy of $z_i \sim \mathcal{N}(\bar{z}, \sigma_i^2)$ is $H(z_i) = \frac{1}{2} \ln(2\pi e \sigma_i^2)$.

The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that

$$I(z_i; s_{ji}) = H(z_i) + H(s_{ji}) - H(z_i, s_{ji}) = \frac{1}{2} \log\left(\frac{\sigma_i^2 \sigma_{sji}^2}{|\Sigma_{z_i s_{ji}}|}\right) = \frac{1}{2} \log\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right),$$

where $|\Sigma_{z_i s_{ji}}| = \sigma_{sji}^2 \sigma_{\delta ji}^2$ is the determinant of the variance-covariance matrix of z_i and s_{ji}.

Hence $I(z_i; s_{ji}) = 0$ if $\sigma_{\delta ji}^2 = \sigma_i^2$.

Across assets, $I(z; s_j) = \sum_{i=1}^{n} I(z_i; s_{ji}) = \frac{1}{2} \sum_{i=1}^{n} \log\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right) = \frac{1}{2} \log\left(\prod_{i=1}^{n} \frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right) \le K_j.$

$\square$

Information Objective. Expected utility is given by

$$E_{0j}[U_j] = \frac{1}{2\rho} E_{0j}\left[\sum_{i=1}^{n} \frac{(\hat{\mu}_{ji} - rp_i)^2}{\hat{\sigma}_{ji}^2}\right] = \frac{1}{2\rho} \sum_{i=1}^{n} \frac{E_{0j}\left[(\hat{\mu}_{ji} - rp_i)^2\right]}{\hat{\sigma}_{ji}^2} = \frac{1}{2\rho} \sum_{i=1}^{n} \left(\frac{\hat{R}_{ji}^2 + \hat{V}_{ji}}{\hat{\sigma}_{ji}^2}\right),$$

where $\hat{R}_{ji}$ and $\hat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, $\hat{\mu}_{ji} - rp_i$. Conjecture (and later verify) that prices are normally distributed, $p_i \sim \mathcal{N}\left(\bar{p}_i, \sigma_{pi}^2\right)$.

$$\hat{R}_{ji} \equiv E_{0j}(\hat{\mu}_{ji} - rp_i) = \bar{z} - r\bar{p}_i,$$

$$\hat{V}_{ji} \equiv V_{0j}(\hat{\mu}_{ji} - rp_i) = Var(\hat{\mu}_{ji}) + r^2\sigma_{pi}^2 - 2rCov(\hat{\mu}_{ji}, p_i).$$

The signal structure implies that $Var(\hat{\mu}_{ji}) = \sigma_{sji}^2$.

Following Admati (1985), we conjecture (and later verify) that prices are $p_i = a_i + b_i\varepsilon_i - c_i\nu_i$, for some coefficients $a_i, b_i, c_i \ge 0$. We compute $Cov(\hat{\mu}_{ji}, p_i)$ exploiting the fact that posterior beliefs and prices are conditionally independent given payoffs. We obtain

$$\hat{V}_{ji} = \sigma_{sji}^2 + r^2\sigma_{pi}^2 - 2rb_i\sigma_{sji}^2 = (1 - rb_i)^2 \sigma_i^2 + r^2 c_i^2 \sigma_x^2 - (1 - 2rb_i)\hat{\sigma}_{ji}^2.$$

Hence the distribution of expected excess returns is normal with mean and variance:

$$\widehat{R}_{ji} = \bar{z} - ra_i \quad \text{and} \quad \widehat{V}_{ji} = (1 - rb_i)^2 \sigma_i^2 + r^2 c_i^2 \sigma_x^2 - (1 - 2rb_i) \widehat{\sigma}_{ji}^2.$$

Expected utility becomes

$$E_{0j}[U_j] = \frac{1}{2\rho} \sum_{i=1}^n G_i \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} - \frac{1}{2\rho} \sum_{i=1}^n (1 - 2rb_i),$$

where $G_i \equiv (1 - rb_i)^2 + \frac{r^2 c_i^2 \sigma_x^2}{\sigma_i^2} + \frac{(\bar{z} - ra_i)^2}{\sigma_i^2}$, and where the second summation is independent of the investor's choices. $\qquad \square$

Proof of Lemma 1 (Information Choice). The linear objective function and the convex constraint imply that each investor allocates all capacity to learning about a single asset. For all other assets, the posterior variance is equal to the prior variance. Let l_j index the asset about which investor j learns. The information constraint becomes $\prod_{i=1}^n \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} = \frac{\sigma_{l_j}^2}{\widehat{\sigma}_{jl_j}^2} = e^{2K_j}$, and hence the variance of the investor's beliefs is given by

$$\widehat{\sigma}_{ji}^2 = \begin{cases} e^{-2K_j} \sigma_i^2 & \text{if } i = l_j, \\ \sigma_i^2 & \text{if } i \neq l_j. \end{cases}$$

The investor's problem becomes picking the asset l_j to maximize $\sum_{i=1}^n G_i \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} = \left(e^{2K_j} - 1\right) G_{l_j} + \sum_{i=1}^n G_i$. Since $e^{2K_j} > 1$, the objective is maximized by allocating all capacity to the asset with the largest utility gain: $l_j \in \arg\max_i G_i$. The distribution of posterior beliefs follows. $\qquad \square$

Conditional Distribution of Signals. Conditional on the realized payoff, the signal is a normally distributed random variable, with mean and variance given by

$$E(s_{ji}|z_i) = \bar{s}_{ji} + \frac{Cov(s_{ji}, z_i)}{\sigma_i^2} (z_i - \bar{z}) = \begin{cases} \bar{z} + \left(1 - e^{-2K_j}\right) \varepsilon_i & \text{if } i = l_j \\ \bar{z} & \text{if } i \neq l_j, \end{cases}$$

$$V(s_{ji}|z_i) = \sigma_{sji}^2 \left(1 - \frac{Cov^2(s_{ji}, z_i)}{\sigma_{sji}^2 \sigma_i^2}\right) = \begin{cases} \left(1 - e^{-2K_j}\right) e^{-2K_j} \sigma_i^2 & \text{if } i = l_j \\ 0 & \text{if } i \neq l_j. \end{cases} \qquad \square$$

Proof of Lemma 2 (Equilibrium Prices). The market clearing condition for each asset in state (z_i, x_i) is

$$\int_{M_{1i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_1} \rho \sigma_i^2}\right) dj + \int_{M_{2i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_2} \rho \sigma_i^2}\right) dj + (1 - m_{1i} - m_{2i}) \left(\frac{\bar{z} - rp_i}{\rho \sigma_i^2}\right) = x_i,$$

where M_{1i} denotes the set of measure $m_{1i} \in [0, \lambda]$ of sophisticated investors who choose to learn about asset i, and M_{2i} denotes the set of measure $m_{2i} \in [0, 1 - \lambda]$, of unsophisticated investors who choose to learn about asset i.

Using the conditional distribution of the signals, $\int_{M_{1i}} s_{ji} dj = m_{1i} \left[\bar{z} + \left(1 - e^{-2K_1}\right) \varepsilon_i\right]$ for the type-1 investors, and analogously for the type-2 investors. Then, the market clearing condition can be written as $\alpha_1 \bar{z} + \alpha_2 \varepsilon_i - x_i = \alpha_1 rp_i$, where

$$\alpha_1 \equiv \frac{1 + m_{1i}(e^{2K_1} - 1) + m_{2i}(e^{2K_2} - 1)}{\rho \sigma_i^2} \text{ and } \alpha_2 \equiv \frac{m_{1i}(e^{2K_1} - 1) + m_{2i}(e^{2K_2} - 1)}{\rho \sigma_i^2}.$$

We obtain identification of the coefficients in $p_i = a_i + b_i \varepsilon_i - c_i \nu_i$ as

$$a_i = \frac{1}{r} \left[\bar{z} - \frac{\bar{x}}{\alpha_1} \right], \ b_i = \frac{\alpha_2}{r \alpha_1}, \text{ and } c_i = \frac{1}{r \alpha_1}.$$

Let $\Phi_i \equiv m_{1i}(e^{2K_1} - 1) + m_{2i}(e^{2K_2} - 1)$ be a measure of the information capacity allocated to learning about asset i in equilibrium. Further substitution yields

$$a_i = \frac{1}{r} \left(\bar{z} - \frac{\rho \sigma_i^2 \bar{x}}{1 + \Phi_i} \right), \quad b_i = \frac{1}{r} \left(\frac{\Phi_i}{1 + \Phi_i} \right), \quad c_i = \frac{1}{r} \left(\frac{\rho \sigma_i^2}{1 + \Phi_i} \right). \qquad \Box$$

Proof of Lemma 3 (Equilibrium Learning). Substituting a_i, b_i, and c_i in $G_i \equiv (1 - rb_i)^2 + \frac{r^2 c_i^2 \sigma_x^2}{\sigma_i^2} + \frac{(\bar{z} - ra_i)^2}{\sigma_i^2}$ and defining $\xi_i \equiv \sigma_i^2 (\sigma_x^2 + \bar{x}^2)$ gives $G_i = \frac{1 + \rho^2 \xi_i}{(1 + \Phi_i)^2}$.

By Lemma 1, each investor learns about a single asset among the assets with the highest gain. WLOG, assets are ordered such that $\sigma_i > \sigma_{i+1}$, for all $i \in \{1, ..., n - 1\}$. First suppose that all investors learn about the same asset. Since G_i is increasing in σ_i, this asset is asset 1. All investors learn about asset 1 as long as $\phi \leq \phi_1 \equiv \sqrt{\frac{1 + \rho^2 \xi_1}{1 + \rho^2 \xi_2}} - 1$. At this threshold, some investors switch and learn about the second asset.

For $\phi > \phi_1$, equilibrium gains must be equated among all assets with positive learning mass. Otherwise, investors have an incentive to switch to learning about the asset with the higher gain. Moreover, the gains of all assets with zero learning mass must be strictly lower. Otherwise, an investor would once again have the incentive to deviate and learn about one of these assets.

To derive expressions for the mass of investors learning about each asset, we assume that the participation of sophisticated and unsophisticated investors in learning about a particular asset is proportional to their mass in the population: $m_{1i} = \lambda m_i$ and $m_{2i} = (1 - \lambda) m_i$, where m_i is the total mass of investors learning about asset i. The necessary and sufficient conditions for determining $\{m_i\}_{i=1}^n$ are $\sum_{i=1}^k m_i = 1$; $\frac{1 + \phi m_i}{1 + \phi m_1} = c_{i1}$, for any $i \in \{2, ..., k\}$, where $c_{i1} \equiv \sqrt{\frac{1 + \rho^2 \xi_i}{1 + \rho^2 \xi_1}} \leq 1$, with equality *iff* $i = 1$; and $m_i = 0$ for any $i \in \{k + 1, ..., n\}$. Recursively, $m_i = c_{i1} m_1 - \frac{1}{\phi}(1 - c_{i1})$, $\forall i \in \{2, ..., k\}$. Using $\sum_{i=1}^k m_i = 1$, and defining $C_k \equiv \sum_{i=1}^k c_{i1}$, we obtain the solution for m_1 given by $m_1 = \frac{1}{C_k} + \frac{1}{\phi} \left(\frac{k}{C_k} - 1 \right)$. Using this expression, we obtain the solution for all m_i, $i \in \{1, ..., k\}$, $m_i = \frac{c_{i1}}{C_k} + \frac{1}{\phi} \left(\frac{k c_{i1}}{C_k} - 1 \right)$. $\qquad \Box$

Proof of Lemma 4 (Learning Dynamics). (i) First, consider a local increase in ϕ to some $\phi' \leq \phi_k$, such that no new assets are learned about in equilibrium (k and C_k are unchanged). For $i \in \{1, ..., k\}$,

$$\frac{dm_i}{d\phi} = -\frac{1}{\phi^2} \left(\frac{k c_{i1}}{C_k} - 1 \right), \text{ where } c_{i1} \equiv \sqrt{\frac{1 + \rho^2 \xi_i}{1 + \rho^2 \xi_1}} \leq 1 \text{ and } C_k \equiv \sum_{i=1}^k c_{i1}.$$

Hence m_i is strictly decreasing in ϕ if $c_{i1} > \frac{C_k}{k}$ (namely, if the asset is above average in terms of relative volatility), and m_i is increasing in ϕ otherwise. Since c_{i1} is decreasing in i, the condition $c_{\bar{\imath}1} = C_k/k$ defines the cutoff asset $\bar{\imath}$. Moreover, note that for $i \in \{1, ..., \bar{\imath}\}$, the absolute value of $\frac{dm_i}{d\phi}$ is decreasing in i, such that the masses of the more volatile assets fall by more than those of the less volatile assets. Likewise, for $i \in \{\bar{\imath}+1, ..., k\}$, the value of $\frac{dm_i}{d\phi}$ is increasing in i, such that the masses of the less volatile assets increase by more than those of the more volatile assets. This results in a flattening of the distribution of investors across assets.

Next, suppose that $k < n$, and consider an increase in ϕ to some $\phi' > \phi_k$, such that $k' > k$ assets are learned about (with $k' \leq n$). Let the new equilibrium masses be denoted by m_i' for $i \in \{1, ..., k'\}$. Hence, $\Sigma_{i=1}^k m_i' < 1$. Using the recursive expression for m_i in terms of m_1, for $i \in \{2, ..., k\}$

$$m_i - m_i' = c_{i1}(m_1 - m_1') - (1 - c_{i1})\left(\frac{1}{\phi} - \frac{1}{\phi'}\right).$$

Suppose that $m_1 \leq m_1'$. Then $\Sigma_{i=1}^k m_i - \Sigma_{i=1}^k m_i' = 1 - \Sigma_{i=1}^k m_i' < 0$, which is a contradiction. Hence $m_1 > m_1'$. Moreover, since c_{i1} is decreasing in i, the condition $m_{\bar{\imath}} = m_{\bar{\imath}}'$ defines the threshold value for c_{i1} that defines the cutoff asset $\bar{\imath}$.

(ii) First, consider a local increase in ϕ to some $\phi' < \phi_k$, such that no new assets are learned about (k and C_k are unchanged). For $i \in \{1, ..., k\}$,

$\frac{d(\phi m_i)}{d\phi} = \frac{c_{i1}}{C_k} > 0.$

Next, suppose that $k < n$, and consider an increase in ϕ to some $\phi' > \phi_k$, such that $k' > k$ assets are learned about in equilibrium (with $k' \leq n$). First, for the new assets that are actively traded, $i \in \{k+1, ..., k'\}$, $m_i' > m_i = 0$, hence, $\phi' m_i' > \phi m_i$. Second, consider an asset $i \in \{1, ..., k\}$ and an asset $h \in \{k+1, ..., k'\}$. Let the new equilibrium gains be denoted by G_i' and G_h'. Then $G_i > G_h$, which implies that $1 + \phi m_i < c_{ih}$, and $G_i' = G_h'$, which implies that $1 + \phi' m_i' = (1 + \phi' m_h')c_{ih} > (1 + \phi' m_h')(1 + \phi m_i) \Leftrightarrow \phi' m_i' > \phi m_i + \phi' m_h'(1 + \phi m_i) > \phi m_i.$

(iii) Let $K_1 = K$ and $K_2 = \delta K$, for some $\delta \in (0,1)$, and consider a symmetric increase in K_j to $(1 + \gamma)K_j$ such that the first $k' \geq k$ assets are learned about.

Let $\bar{\imath}$ denote the cutoff asset determined in part (i). For $i \in \{\bar{\imath}, ..., k'\}$, both $m_i(e^{2K_1} - 1)$ and $m_i(e^{2K_2} - 1)$ increase, since $m_i' \geq m_i$, but $m_i(e^{2K_1} - 1)$ grows by more since e^x is convex.

For $i \in \{1, ..., \bar{\imath} - 1\}$, m_i is decreasing in ϕ.
Let $m_{i\phi} \equiv \frac{dm_i}{d\phi}$. The derivatives of interest are

$$D_1 \equiv \frac{d[m_i(e^{2K_1} - 1)]}{dK_1} = m_{i\phi}(e^{2K} - 1)\frac{d\phi}{dK} + 2e^{2K}m_i$$

$$D_2 \equiv \frac{d[m_i(e^{2K_2} - 1)]}{dK_2} = m_{i\phi}(e^{2K\delta} - 1)\frac{d\phi}{dK} + 2\delta e^{2K\delta}m_i$$

where $\frac{d\phi}{dK} = 2\lambda e^{2K} + 2\delta(1-\lambda)e^{2K\delta} > 0$.

Factoring out $2e^{2K}$ yields

$$
\begin{aligned}
D_1 &= 2e^{2K}\left\{m_i + m_{i\phi}\left(e^{2K}-1\right)\left[\lambda + (1-\lambda)\delta e^{2K(\delta-1)}\right]\right\} \\
&= 2e^{2K}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + (1-\lambda)\delta\left(e^{2K\delta}-e^{2K(\delta-1)}\right)\right]\right\} \\
&\overset{(1)}{>} 2e^{2K}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + (1-\lambda)\left(e^{2K\delta}-1\right)\right]\right\} \\
&= 2e^{2K}\left\{m_i + m_{i\phi}\phi\right\} = 2e^{2K}\left[\tfrac{d(\phi m_i)}{d\phi}\right] \overset{(2)}{>} 0,
\end{aligned}
$$

where (2) follows from part (ii) above and (1) follows from the evaluation of the function

$F(\delta) = e^{2K\delta} - 1 - \delta\left(e^{2K\delta} - e^{2K(\delta-1)}\right)$ for which $F(0) = F(1) = 0$, $F'(\delta) = 0$ has a unique solution, $F'(\delta) > 0$ and $F'(\delta) < 0$, which imply that $F(\delta) > 0$.

Next, note that $\lambda D_1 + (1-\lambda)D_2 = \left[\tfrac{d(\phi m_i)}{d\phi}\right]\tfrac{d\phi}{dK} = 2\left[\tfrac{d(\phi m_i)}{d\phi}\right]\left[\lambda e^{2K} + \delta(1-\lambda)e^{2K\delta}\right]$. We have just shown that $D_1 > 2e^{2K}\left[\tfrac{d(\phi m_i)}{d\phi}\right]$, so for the equality to hold it must be the case that $D_2 < 2\delta e^{2K\delta}\left[\tfrac{d(\phi m_i)}{d\phi}\right]$. Hence $D_1 > 0$ and $D_1 > D_2$. It remains to be determined if $D_2 > 0$ as well.

$$
\begin{aligned}
D_2 &= 2e^{2K\delta}\left\{\delta m_i + m_{i\phi}\left(1-e^{-2K\delta}\right)\left[\lambda e^{2K} + \delta(1-\lambda)e^{2K\delta}\right]\right\} \\
&= 2e^{2K\delta}\left\{\delta m_i + m_{i\phi}\left[\lambda e^{2K} + \delta(1-\lambda)e^{2K\delta} - \lambda e^{2K-2K\delta} - \delta(1-\lambda)\right]\right\} \\
&= 2e^{2K\delta}\left\{\delta m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) - \lambda\left(e^{2K-2K\delta}-1\right) + \delta(1-\lambda)\left(e^{2K\delta}-1\right)\right]\right\} \\
&> 2e^{2K\delta}\left\{\delta m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + \delta(1-\lambda)\left(e^{2K\delta}-1\right)\right]\right\} \\
&> 2e^{2K\delta}\left\{\delta m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + (1-\lambda)\left(e^{2K\delta}-1\right)\right]\right\} \\
&= 2e^{2K\delta}\left\{\delta m_i + m_{i\phi}\phi\right\} = 2e^{2K\delta}\left[\tfrac{d(\phi m_i)}{d\phi}\right] - (1-\delta)m_i.
\end{aligned}
$$

Hence, if δ is not too small (i.e. capacity dispersion is not too large), then $D_2 > 0$ for $i \in \{1, ..., \bar{\imath} - 1\}$ as well.

Hence, for assets $i \in \{1, ..., \bar{\imath} - 1\}$, for which the mass of investors falls in response to the capacity growth, $m_i\left(e^{2K_1}-1\right)$ grows and $m_i\left(e^{2K_2}-1\right)$ grows by less, or even falls, if capacity dispersion is large enough. $\qquad\square$

Analytic Results

Proof of Proposition 1. Results follow from equations (14-16). $\qquad\square$

Proof of Proposition 2. **(i)** Follows from the definition of capital income per capita and equation (15). **(ii)** Since for all $i \in \{1, ..., k\}$, the gains G_i are equated in equilibrium, then $E\left[\pi_{1i} - \pi_{2i}\right]$ is increasing in m_i, which in turn is increasing in σ_i^2. $\qquad\square$

Proof of Proposition 3. **(i)** The increase in dispersion keeps ϕ unchanged. Therefore, using equation (11), the masses m_i are unchanged. With both ϕ and m_i unchanged, prices

are unchanged. **(ii)** The result follows from equation (15): masses and prices do not change, and dispersion, $\left(e^{2K_1} - e^{2K_2}\right)$ increases. **(iii)** Relative capital income is

$$\frac{\pi_{1i}}{\pi_{2i}} = \frac{(\bar{z}_i - rp_i)(z_i - rp_i) + \left(e^{2K_1} - 1\right)m_i(z_i - rp_i)^2}{(\bar{z}_i - rp_i)(z_i - rp_i) + \left(e^{2K_2} - 1\right)m_i(z_i - rp_i)^2} > 1.$$

Since prices are unchanged, $(\bar{z}_i - rp_i)(z_i - rp_i)$ and $m_i(z_i - rp_i)^2$ are unchanged. Since $K_1' > K_1$ and $K_2' < K_2$, the second term in π_{1i} increases and the second term in π_{2i} decreases. $\qquad\square$

Proof of Proposition 4. **(i)** Using equilibrium prices, $\bar{p}_i = \frac{1}{r}\left(\bar{z} - \frac{\rho\sigma_i^2\bar{x}}{1+\phi m_i}\right)$. Per Lemma 4, ϕm_i is increasing in ϕ. Hence, for $i \in \{1,...,k\}$, $\bar{p}_i$ is increasing in ϕ. The result for equilibrium expected excess returns $E[z_i - r\bar{p}_i]$ follows.

(ii) Since $\lambda E[q_{1i}] + (1-\lambda)E[q_{2i}] = \bar{x}$, it is sufficient to show that for $i \in \{1,...,k'\}$, $E[q_{1i}]$ increases in response to symmetric capacity growth. Let $K \equiv K_1$, and $K_2 = \delta K$, with $\delta \in (0,1)$. Since

$$E[q_{1i}] = \frac{1+m_i\left(e^{2K}-1\right)}{(1+\phi m_i)}\bar{x}, \text{ then } \frac{dE[q_{1i}]}{dK} = \frac{\bar{x}}{(1+\phi m_i)^2}\left[\frac{d[m_i(e^{2K}-1)]}{dK}(1+\phi m_i) - \frac{d(\phi m_i)}{d\phi}\frac{d\phi}{dK}m_i\left(e^{2K}-1\right)\right].$$

Hence $sign\left(\frac{dE[q_{1i}]}{dK}\right) = sign\left(\frac{d[m_i(e^{2K}-1)]}{dK} - \frac{d(\phi m_i)}{d\phi}\frac{d\phi}{dK}\frac{m_i(e^{2K}-1)}{1+\phi m_i}\right).$

In the proof of Lemma 4, we show that $\frac{d[m_i(e^{2K}-1)]}{dK} > 2e^{2K}\frac{d(\phi m_i)}{d\phi} > 0$. Hence,

$$sign\left(\frac{dE[q_{1i}]}{dK}\right) = sign\left(2e^{2K} - \frac{d\phi}{dK}\frac{m_i(e^{2K}-1)}{1+\phi m_i}\right)$$

$$= sign\left(2e^{2K} - \frac{2m_i\left[\lambda e^{2K}+(1-\lambda)\delta e^{2K\delta}\right]\left(e^{2K}-1\right)}{1+m_i\left[\lambda(e^{2K}-1)+(1-\lambda)\left(e^{2K\delta}-1\right)\right]}\right)$$

$$= sign\left(e^{2K} - \left(e^{2K}-1\right)\frac{m_i\left[\lambda e^{2K}+(1-\lambda)\delta e^{2K\delta}\right]}{1+m_i\left[\lambda e^{2K}+(1-\lambda)e^{2K\delta}\right]-m_i}\right)$$

$$\stackrel{(1)}{=} sign\left(e^{2K} - \left(e^{2K}-1\right)\left[\frac{m_i\left[\lambda e^{2K}+(1-\lambda)e^{2K\delta}\right]}{1+m_i\left[\lambda e^{2K}+(1-\lambda)e^{2K\delta}\right]-m_i}\right]\right)$$

$$\stackrel{(2)}{=} sign\left(e^{2K} - \left(e^{2K}-1\right)\right) > 0$$

where (1) follows from $\delta \in (0,1)$, and (2) follows from the fact that the term in square brackets is less than 1.

(iii) Let the per capita capital income be decomposed into a component C_i that is common across investor groups, and a component that is group-specific:
$\pi_{1i} = c_i + \frac{1}{\rho\sigma_i^2}m_i\left(e^{2K}-1\right)(z_i - rp_i)^2$, where $c_i \equiv \frac{1}{\rho\sigma_i^2}(\bar{z} - rp_i)(z_i - rp_i)$, with expected value C_i. Then $E[\pi_{1i}] = C_i + \frac{1}{\rho\sigma_i^2}m_i\left(e^{2K}-1\right)E\left[(z_i - rp_i)^2\right] = C_i + \frac{1}{\rho}m_i\left(e^{2K}-1\right)G_i$, where G_i is the gain from learning about asset i, equated across all $i \in \{1,...,k\}$.

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We then obtain that $\frac{E[\pi_{1i}]}{E[\pi_{2i}]} = \frac{C_i + \frac{1}{\rho}m_i\left(e^{2K}-1\right)G_i}{C_i + \frac{1}{\rho}m_i\left(e^{2K\delta}-1\right)G_i}$.

In response to an increase in K, C_i and G_i decrease, but they affect both sophisticated and unsophisticated profits in the same way. From Lemma 4, $m_i\left(e^{2K}-1\right)$ increases by more than $m_i\left(e^{2K\delta}-1\right)$ in response to a change in K. Hence overall, $\frac{E[\pi_{1i}]}{E[\pi_{2i}]}$ increases.

$\square$

Derivation of volume per capita. We define the volume of trade in asset i between two periods, across all optimizing investors j in group g as $V_i^g \equiv \int \left| q'_{ji} - q_{ji} \right| dj$. Integrating over all possible realizations of q'_{ji} and q_{ji}, we obtain average volume across many periods, $\overline{V}_i^g$. We assume that investors do not change groups over time. To ease notation, most of the derivation omits group and asset superscripts.

Volume between two periods for a generic group First, we calculate the expected volume of trade for each asset by agents in each group from period t to $t+1$. Let f and F denote the pdf and cdf of current holdings, with mean μ_q and standard deviation σ_q. Let f' and F' denote the pdf and cdf of future holdings, with mean μ'_q and standard deviation σ'_q.

STEP 1. Consider a particular investor with holdings q in the current period. The investor's expected volume of trade between the current and the next period is

$$v\left(q\right) \equiv \int_{-\infty}^{+\infty} |q' - q| \, f'\left(q'\right) dq' = 2qF\left(q\right) - q - 2F'\left(q\right) E_{f'}\left[q'|q' < q\right] + \mu'_q.$$

Using the formula for the expected value of a normal truncated from above,

$$v\left(q\right) = 2qF\left(q\right) - q - 2\mu'_q F'\left(q\right) + 2\sigma'^2_q f'\left(q\right) + \mu'_q.$$

STEP 2. Integrating over the (normal) distribution of holdings q in the group,

$$V^g = \int_{-\infty}^{+\infty} v\left(q\right) f\left(q\right) dq = 2\int_{-\infty}^{+\infty} qF\left(q\right) f\left(q\right) dq - \mu_q - 2\mu'_q \int_{-\infty}^{+\infty} F'\left(q\right) f\left(q\right) dq + 2\sigma'^2_q \int_{-\infty}^{+\infty} f'\left(q\right) f\left(q\right) dq + \mu'_q.$$

Using the formulas $\int_{-\infty}^{+\infty} \exp\left\{-ax^2 + bx + c\right\} dx = \sqrt{\frac{\pi}{a}} \exp\left\{\frac{b^2}{4a} + c\right\}$,

$$\int_{-\infty}^{+\infty} \Phi\left(a + bx\right) \phi\left(x\right) dx = \Phi\left(\frac{a}{\sqrt{1+b^2}}\right) \text{ and } \int_{-\infty}^{+\infty} x\Phi\left(bx\right) \phi\left(x\right) dx = \frac{b}{\sqrt{2\pi(1+b^2)}},$$

we compute $J_1 \equiv \int_{-\infty}^{+\infty} qF\left(q\right) f\left(q\right) dq = \frac{\mu_q}{2} + \frac{\sigma_q}{2\sqrt{\pi}}$,

$$J_2 \equiv \int_{-\infty}^{+\infty} F'\left(q\right) f\left(q\right) dq = \Phi\left(\frac{\mu_q - \mu'_q}{\sqrt{\sigma_q^2 + \sigma_q'^2}}\right),$$

$$J_3 \equiv \int_{-\infty}^{+\infty} f'(q) f(q) \, dq = \frac{1}{\sqrt{2\pi(\sigma_q^2 + \sigma_q'^2)}} \exp\left\{ -\frac{(\mu_q - \mu_q')^2}{2(\sigma_q^2 + \sigma_q'^2)} \right\}.$$

Hence $V^g = \frac{\sigma_q}{\sqrt{\pi}} - 2\mu_q' \Phi\left(\frac{\mu_q - \mu_q'}{\sqrt{\sigma_q^2 + \sigma_q'^2}} \right) + \frac{2\sigma_q'^2}{\sqrt{2\pi(\sigma_q^2 + \sigma_q'^2)}} \exp\left\{ -\frac{(\mu_q - \mu_q')^2}{2(\sigma_q^2 + \sigma_q'^2)} \right\} + \mu_q'$, where the means and standard deviations are group and asset specific.

Since the shocks are i.i.d., holdings have the same cross-sectional variance in all periods, $\sigma_q' = \sigma_q$, though they will have different means, depending on shock realizations. Hence

$$V^g = \frac{\sigma_q}{\sqrt{\pi}} \left[1 + \exp\left\{ -\frac{(\mu_q - \mu_q')^2}{4\sigma_q^2} \right\} \right] + \mu_q' \left[1 - 2\Phi\left(\frac{\mu_q - \mu_q'}{\sqrt{2\sigma_q^2}} \right) \right].$$

Average volume across many periods for a generic group   We assume no change in the environment, including no change in capacities and hence learning. Let the distribution of mean holdings μ_q be denoted by g, with mean and variance μ_μ and σ_μ^2. Since the shocks are normal i.i.d., this distribution is stationary and also normal. The average volume across all possible realizations of μ_q and μ_q' is

$$\overline{V}^g = \int_{-\infty}^{+\infty} v^g(\mu_q) \, g(\mu_q) \, d\mu_q, \text{ with } v^g(\mu_q) = \int_{-\infty}^{+\infty} V^g(\mu_q, \mu_q') \, g(\mu_q') \, d\mu_q'.$$

STEP 3. Using the expression for V^g,

$$v^g(\mu_q) = \frac{\sigma_q}{\sqrt{\pi}} + \frac{\sigma_q}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \exp\left\{ -\frac{(\mu_q - \mu_q')^2}{4\sigma_q^2} \right\} g(\mu_q') \, d\mu_q' + \mu_\mu - 2\int_{-\infty}^{+\infty} \mu_q' \Phi\left(\frac{\mu_q - \mu_q'}{\sqrt{2\sigma_q^2}} \right) g(\mu_q') \, d\mu_q'$$

Using the formulas for integrals of normal distributions, we compute

$$J_1 \equiv \int_{-\infty}^{+\infty} \exp\left\{ -\frac{(\mu_q - \mu_q')^2}{4\sigma_q^2} \right\} g(\mu_q') \, d\mu_q' = \sqrt{\frac{2\sigma_q^2}{\sigma_\mu^2 + 2\sigma_q^2}} \exp\left\{ -\frac{(\mu_q - \mu_\mu)^2}{2(\sigma_\mu^2 + 2\sigma_q^2)} \right\},$$

$$J_2 \equiv \int_{-\infty}^{+\infty} \mu_q' \Phi\left(\frac{\mu_q - \mu_q'}{\sqrt{2\sigma_q^2}} \right) g(\mu_q') \, d\mu_q' = \mu_\mu \Phi\left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right) - \frac{\sigma_\mu^2}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \phi\left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right).$$

Then $v^g(\mu_q) = \frac{\sigma_q}{\sqrt{\pi}} + \frac{\sigma_q}{\sqrt{\pi}} J_1 + \mu_\mu - 2J_2$ becomes

$$\frac{\sigma_q}{\sqrt{\pi}} + \frac{\sigma_q^2 \sqrt{2}}{\sqrt{\pi(\sigma_\mu^2 + 2\sigma_q^2)}} \exp\left\{ -\frac{(\mu_q - \mu_\mu)^2}{2(\sigma_\mu^2 + 2\sigma_q^2)} \right\} + \mu_\mu - 2\mu_\mu \Phi\left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right) + \frac{2\sigma_\mu^2}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \phi\left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right).$$

STEP 4. Finally, integrating $v^g(\mu_q)$ over all possible realizations of μ_q, we obtain

$$\overline{V}^g = \frac{\sigma_q}{\sqrt{\pi}} + \frac{\sigma_q^2 \sqrt{2}}{\sqrt{\pi(\sigma_\mu^2 + 2\sigma_q^2)}} \int_{-\infty}^{+\infty} \exp\left\{ -\frac{(\mu_q - \mu_\mu)^2}{2(\sigma_\mu^2 + 2\sigma_q^2)} \right\} g(\mu_q) \, d\mu_q + \mu_\mu$$

$$-2\mu_\mu \int_{-\infty}^{+\infty} \Phi\left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right) g(\mu_q) \, d\mu_q + \frac{2\sigma_\mu^2}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \int_{-\infty}^{+\infty} \phi\left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} \right) g(\mu_q) \, d\mu_q.$$

We compute

$$J_1 \equiv \int_{-\infty}^{+\infty} \exp\left\{-\frac{(\mu_q - \mu_\mu)^2}{2(\sigma_\mu^2 + 2\sigma_q^2)}\right\} g(\mu_q) \, d\mu_q = \sqrt{\frac{\sigma_\mu^2 + 2\sigma_q^2}{2(\sigma_\mu^2 + \sigma_q^2)}},$$

$$J_2 \equiv \int_{-\infty}^{+\infty} \Phi\left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}}\right) g(\mu_q) \, d\mu_q = \frac{1}{2},$$

$$J_3 \equiv \int_{-\infty}^{+\infty} \phi\left(\frac{\mu_q - \mu_\mu}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}}\right) g(\mu_q) \, d\mu_q = \frac{J_1}{\sqrt{2\pi}}.$$

Then

$$\overline{V}^g = \frac{\sigma_q}{\sqrt{\pi}} + \frac{\sigma_q^2 \sqrt{2}}{\sqrt{\pi(\sigma_\mu^2 + 2\sigma_q^2)}} J_1 + \mu_\mu - 2\mu_\mu J_2 + \frac{2\sigma_\mu^2}{\sqrt{\sigma_\mu^2 + 2\sigma_q^2}} J_3 = \frac{1}{\sqrt{\pi}}\left(\sigma_q + \sqrt{\sigma_q^2 + \sigma_\mu^2}\right).$$

Variances by Investor Group

Consider the groups $g = SL, UL$ of sophisticated and unsophisticated investors who learn about asset i. These groups differ in their capacities only. A particular investor j in group g holds $q_{ji} = e^{2K_g}(s_{ji} - rp_i)/\rho\sigma_i^2$. The cross-sectional variance of holdings for this group, conditional on the realized shocks, is

$$\left(\sigma_{qi}^g\right)^2 = \left(\frac{e^{4K_g}}{\rho^2\sigma_i^4}\right) Var\left(s_{ji} - rp_i\right) = \frac{e^{2K_g} - 1}{\rho^2\sigma_i^2}.$$

The cross-sectional mean is

$$\mu_{qi}^g = \left(\frac{e^{2K_g}}{\rho\sigma_i^2}\right) E\left(s_{ji} - rp_i\right) = \frac{e^{2K_g}}{1 + \phi m_i}\left(\overline{x} + \nu_i\right) + \frac{e^{2K_g} - 1 - \phi m_i}{\rho\sigma_i^2(1 + \phi m_i)}\varepsilon_i.$$

The expected value of mean of holdings is $\mu_{\mu i}^g = \frac{e^{2K_g}}{1 + \phi m_i}\overline{x}$ and the variance of mean holdings is

$$\left(\sigma_{\mu i}^g\right)^2 = \left(\frac{e^{2K_g}}{1 + \phi m_i}\right)^2 \sigma_x^2 + \left(\frac{e^{2K_g} - 1 - \phi m_i}{1 + \phi m_i}\right)^2 \frac{1}{\rho^2\sigma_i^2}.$$

Consider the group NL of investors who are not learning about asset i. All investors in this group hold the same quantity $q_{ji} = \mu_{qi} = (\overline{z} - rp_i)/\rho\sigma_i^2$. Hence

$$\left(\sigma_{qi}^{NL}\right)^2 = 0 \text{ and } \mu_{qi}^{NL} = \frac{1}{\rho\sigma_i^2}\left(\overline{z} - rp_i\right).$$

The mean and variance of mean holdings are

$$\mu_{\mu i}^{NL} = \frac{\overline{x}}{1 + \phi m_i} \text{ and } \left(\sigma_{\mu i}^{NL}\right)^2 = \left(\frac{1}{1 + \phi m_i}\right)^2 \sigma_x^2 + \left(\frac{\phi m_i}{1 + \phi m_i}\right)^2 \frac{1}{\rho^2\sigma_i^2}.$$

Consider the assets with zero learning, ZL. For assets that are not learned about by anyone ($m_i = 0$, $\phi m_i = 0$), all investors hold $q_{ji} = \mu_{qi} = (\overline{z} - rp_i)/\rho\sigma_i^2$. Hence

$$\left(\sigma_{qi}^{ZL}\right)^2 = 0 \text{ and } \mu_{qi}^{ZL} = \frac{1}{\rho\sigma_i^2}(\overline{z} - rp_i).$$

The mean and variance of mean holdings are

$$\mu_{\mu i}^{ZL} = \overline{x} \text{ and } \left(\sigma_{\mu i}^{ZL}\right)^2 = \sigma_x^2.$$

$\square$

Proof of Proposition 5. First, average volume of active investors, $g = SL, UL$, is

$$\overline{V}^g = \frac{1}{\sqrt{\pi\rho^2\sigma_i^2}}\left[\sqrt{e^{2K_g} - 1} + \sqrt{e^{2K_g} - 1 + \left(\frac{e^{2K_g}}{1+\phi m_i}\right)^2 \rho^2\sigma_i^2\sigma_x^2 + \left(\frac{e^{2K_g}-1-\phi m_i}{1+\phi m_i}\right)^2}\right]$$

$\overline{V}^g$ is increasing in K_g hence $\overline{V}^{SL} > \overline{V}^{UL}$.
Next, average volume of passive investors in actively traded assets is

$$\overline{V}_i^{NL} = \frac{\sigma_\mu^{NL}}{\sqrt{\pi}} = \frac{1}{\sqrt{\pi\rho^2\sigma_i^2}}\sqrt{\left(\frac{\rho\sigma_i\sigma_x}{1+\phi m_i}\right)^2 + \left(\frac{\phi m_i}{1+\phi m_i}\right)^2}.$$

Using $\sqrt{a} + \sqrt{b} > \sqrt{a+b}$,

$$\overline{V}^{UL} > \frac{1}{\sqrt{\pi\rho^2\sigma_i^2}}\left[\sqrt{2\left(e^{2K_2} - 1\right) + \left(\frac{e^{2K_2}}{1+\phi m_i}\right)^2 \rho^2\sigma_i^2\sigma_x^2 + \left(\frac{e^{2K_2}-1-\phi m_i}{1+\phi m_i}\right)^2}\right]$$

$$= \frac{1}{\sqrt{\pi\rho^2\sigma_i^2}}\sqrt{\frac{(\phi m_i)^2\left(2e^{2K_2}-1\right)+2\phi m_i\left(e^{2K_2}-1\right)+\left(e^{4K_2}-1\right)}{(1+\phi m_i)^2} + \left(\frac{e^{2K_2}}{1+\phi m_i}\right)^2 \rho^2\sigma_i^2\sigma_x^2}$$

$$> \frac{1}{\sqrt{\pi\rho^2\sigma_i^2}}\sqrt{\frac{(\phi m_i)^2}{(1+\phi m_i)^2} + \left(\frac{1}{1+\phi m_i}\right)^2 \rho^2\sigma_i^2\sigma_x^2} = \overline{V}_i^{NL}.$$

Hence for $i \in \{1, ..., k\}$, $\overline{V}_i^{SL} > \overline{V}_i^{UL} > \overline{V}_i^{NL}$.

$\square$